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Probabilistic Modeling and Analysis of High-Speed Rough-Terrain Mobile Robots

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Abstract—**Mobile robots have important applications in high speed, rough-terrain scenarios. It would be desirable to construct accurate models of these systems. However, due to the system complexity, accurate modeling is difficult. In this paper a high-speed rough-terrain robot model is presented. Experiments show that this model can accurately predict robot performance in simple, well-known terrain. However in unstructured, rough terrain, performance prediction is less accurate. A stochastic method for analyzing system performance in spite of model parameter uncertainty is presented. A method for studying model sensitivity to parameter uncertainty is also presented. It is shown that stochastic analysis can be used effectively for model-based analysis of real-world rough-terrain robotic systems.**

Keywords-Mobile Robots, Robot Modeling, Rough Terrain

I. INTRODUCTION

Mobile robots are becoming increasingly important in fields that require operation in rough terrain. Planetary exploration, mining, forestry, and hazardous site inspection are a few applications of these systems [1–4]. Military applications are also rapidly gaining importance [5, 6]. Potential missions include logistics, surveillance, fire missions, and soldier assistance. In order to operate effectively in the field, mobile robots must be able to traverse rough, unstructured terrain, often at high speeds.

A fundamental problem associated with such systems is developing accurate models. Accurate models are valuable for design studies, performance prediction, and on-line control purposes. However, constructing an accurate model is difficult. This is due to complex phenomena such as wheel slip, ballistic motion, high-order dynamic effects, and wheel-terrain interaction. These models often contain substantial uncertainty, since it is unlikely that all model parameters (especially those of the terrain) will be precisely known.

The motivation of this work is to develop a stochastic modeling and analysis method for high-speed mobile robots operating on rough terrain. It has been shown that detailed models can accurately predict high-speed robot performance in simple, well-characterized terrain [7]. However, on rough, unknown terrain, model fidelity degrades due to imprecise knowledge of terrain parameters. Note that we are not concerned with studying simplified, low-order models often employed for control system development, as these models usually neglect important effects such as wheel slip and ballistic behavior [8–12]. While this may be allowable for control system development, it is not suitable for performance analysis and prediction.

The proposed analysis method predicts system performance in spite of unknown or variable terrain and robot model parameters. The method employs a sampling method over the space of possible model parameter sets to compute expected robot performance metrics [13]. Sampling-based methods have been used to investigate the performance of complex systems and processes such as nuclear reactor safety, disposal of radioactive waste, environmental risk assessment, and petroleum exploration [14–17]. A method is also presented that computes the model sensitivity parameter uncertainty. These methods can be used as analysis tools for evaluating robot performance off-line. They can also be used as a basis for on-line planning and control algorithms. This work essentially proposes an alternative to deterministic modeling of rough-terrain high-speed mobile robots, due to the substantial uncertainty in such systems.

II. ROBOT AND TERRAIN MODEL

A 15 DOF model of a four-wheeled robot was developed that considers roll, pitch, yaw, suspension dynamics, tire dynamics, and a simplified model of wheel-terrain interaction. This model has been experimentally validated in simple, well-characterized terrain [7, 18]. This model is a lumped parameter representation that includes mass and inertial properties of the robot body and wheels, and compliance and damping of the suspension and tires. A schematic is shown in Figure 1.

Three translational and three rotational degrees-of-freedom (X_b, Y_b, Z_b, Φ, Θ, Ψ, respectively) are assigned to the robot body at the center of mass. The lumped body mass properties includes all suspension components. Suspension spring and damper elements are assumed to be linear functions of displacement and velocity, respectively. The robot's wheels are mounted to the body through independent spring-damper suspensions. Each wheel has a rotational degree-of-freedom for rolling (θ), and a translational degree-of-freedom for motion relative to the body along the suspension travel direction (y). Tire compliance and damping are modeled as a parallel linear spring-damper acting normal to the local ground contact patch. Rolling resistance of the tires is neglected for

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simplicity. Robot command inputs are the torques applied to the rear wheels, (τ) and the steering angle of the front wheels, θ_{steer}.

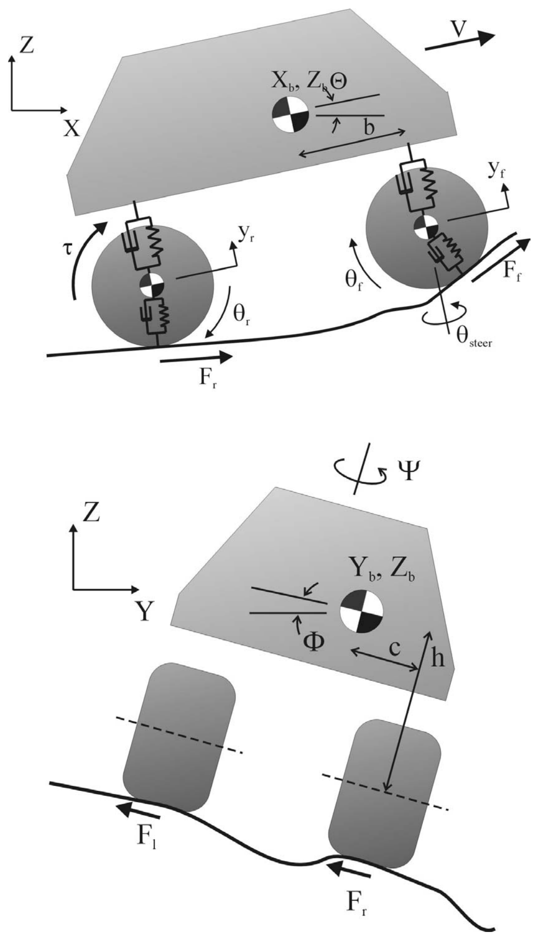

Figure 1. Schematic of high-speed mobile robot model.

Fractal techniques are used to generate terrain geometry [19]. It has been shown that fractals can be used to accurately describe natural terrain [20]. Tractive and braking forces are modeled using a simple force coefficient model, for simplicity in illustrating the analysis approach. Here the terrain is assumed to be rigid and the tires deformable. This is reasonable for robots with pneumatic tires operating in a variety of natural terrains.

Due to the difficultly of analytically modeling such phenomena as wheel slip and ballistic behavior, the analysis of this model was performed in the commercial dynamic analysis software package ADAMS 12.0.

III. Stochastic Uncertainty Analysis

Uncertain knowledge of model parameters can lead to error in predicting a robot's dynamic response. In real-world applications, terrain model uncertainty stems from lack of sensor data, naturally-occurring spatial variation, and/or error inherent in terrain characterization techniques. Robot model uncertainty stems from imprecise parameter measurement, and parameter dependence on time-varying phenomena (such as fuel consumption). It would be desirable to quantify the relationship between parameter uncertainty and robot dynamic response. This would lead to a robust evaluation of robot performance under realistic conditions. Such information could be used as a basis for design studies, and control and planning algorithms that are robust to uncertainty.

Here we present a stochastic method for analyzing the effect of model parameter uncertainty on mobile robot maneuvers. It is assumed that model parameters are normally distributed random variables. The normal distribution could result from physical measurement error, physical property variation, or sensing errors. In this analysis, parameter uncertainty is described as:

$$u = \frac{3\sigma_i}{\bar{x}_i} \tag{1}$$

where σ_i is the standard deviation and $\bar{x}_i$ is the mean of the i^{th} parameter.

The analysis methodology is as follows:

1. Determine vectors composed of the nominal values $\bar{\mathbf{x}}_{in}$ and corresponding standard deviations σ_{in} for all model parameters. This is the input parameter space. Note that uncertainty estimates are derived from engineering judgment, and might depend on physical measurement precision, expected environmental variation for a given task, and sensor uncertainty. Further, this uncertainty can be viewed as a measure of model complexity for phenomena that are measurable but difficult to characterize. This approach is based on the observation that while precise model parameter values are often unknown known, it is usually possible to intelligently bound the uncertainty on such parameters.

2. Randomly sample a set of parameter values $\mathbf{x}_{in}$ from the input parameter space. In this work we employed Matlab's random number generator for sampling purposes.

3. Evaluate the model response using the sampled parameter values $\mathbf{x}_{in}$ and record an output metric $\mathbf{y}_{out}$. This metric is generally related to a particular robot maneuver, such as stopping distance or path tracking error.

4. Repeat steps 2) and 3) *n* times to generate an output distribution with *n* points. Here *n* should be chosen to be large enough such that the output distribution's 99% confidence interval converges.

5. Estimate the output distribution's 99% confidence interval.

This approach results in an output distribution related to a particular maneuver, corresponding to a given set of model parameters and uncertainty levels. Such an approach could be used, for example, to study the expected stopping distance distribution for a particular mobile robot traversing terrain with partially-known characteristics. It could also be used to predict the probability of successful obstacle traversal. Such results would be useful for off-line performance analysis. The result of a suite of such analyses could be stored on-line as part of a motion planning or control algorithm. This is an area of current research.

IV. Sensitivity Analysis

The previous section addressed the effect of model parameter uncertainty on robot dynamic response. It would be desirable to quantify the dynamic response's sensitivity to variation in individual system parameters. This would be useful for understanding the primary causes of response variation. It would also suggest where to direct modeling effort to reduce this variation. This section presents a variance-based method for quantifying the sensitivity of a robot's dynamic response to variation in individual system parameters.

For simple, low-order systems, analytical techniques can be used to compute sensitivity. However the high-speed mobile robot model considered in this work is complex and highly nonlinear, and thus other techniques must be applied. The sensitivity method employed here is a variance-based scheme known as the method of Sobol [21]. It decomposes output variance into components based on Sobol's functional description [22]. Parameter sensitivity is then computed from the partial variances. Total sensitivity indices are computed to estimate individual parameter sensitivity including nonlinear and interaction terms. The Sobol method employs Monte Carlo methods to estimate these terms with a relatively small set of model evaluations [23].

Specifically, the total number of model evaluations N needed to estimate the total sensitivity indices using this method is $N = n(k+1)$, where n is the number of samples used to estimate an integral and k is the number of model parameters. By contrast, for a brute force approach with r levels the number of model evaluations is $N = r^k$. For good accuracy with the Sobol method approximately n = 1000 samples should be used. With a brute force approach a minimum of r = 3 levels should be used. For k = 11 robot model parameters, the total number of model evaluations for each method is (N_{Sobol} = 12000, $N_{factorial}$ = 177147). Thus the Sobol method is substantially more efficient in estimating nonlinear and interaction effects than the full-factorial approach for large parameter sets, and makes the analysis computationally feasible.

To apply Sobol's method, the function input space is first defined over the unit (hyper)cube, i.e. the region

$$\Omega^k = (\mathbf{x} \mid 0 \le x_i \le 1;\;\; i = 1,\ldots,k). \quad (2)$$

where k is the number of parameters. The function $f(\mathbf{x})$ is decomposed into summands of increasing dimensionality as:

$$f(x_1,\ldots,x_k) = f_0 + \sum_{i=1}^{k} f_i(x_i) + \sum_{1\le i<j\le k} f_{ij}(x_i, x_j) + \cdots + f_{1,\ldots,k}(x_1,\ldots,x_k) \,. \quad (3)$$

This decomposition is based on a general representation using multiple integrals, and has been shown to be unique [22]. The total variance D of $f(\mathbf{x})$ is

$$D = \int_{\Omega^k}^{1} f^2(\mathbf{x})d\mathbf{x} - f_0^2 \,. \quad (4)$$

The partial variances due to each term in Equation (3) are computed as

$$D_{i_1,\ldots,i_s} = \int_0^1 \cdots \int_0^1 f_{i_1,\ldots,i_s}^2 (x_{i_1},\ldots x_{i_s}) dx_{i_1},\ldots,dx_{i_s} \,. \quad (5)$$

The sensitivity index for each term in Equation (3) is then defined to be

$$S_{i_1,\ldots,i_s} = \frac{D_{i_1,\ldots,i_s}}{D} \,. \quad (6)$$

The sensitivity indices compute the relative importance of a parameter, or interaction of parameters, as compared to the whole. Complete characterization of a system requires computation of 2^k - 1 sensitivity indices. However, the total sensitivity index can be defined as the sum of all sensitivity indices involving the parameter in question. The total sensitivity index is defined as

$$TS(i) = 1 - \frac{D_{\sim i}}{D} \qquad i = 1,\ldots,k \quad (7)$$

where $D_{\sim i}$ is the variance complement to x_i. The total sensitivity index estimates the overall effects of the i^{th} parameter, including first and higher order terms. The integrals required to compute the total sensitivity indices are estimated using Monte Carlo integration [24]. A total of k + 1 Monte Carlo integral computations are required, with n model evaluations per integral. For the model used here, with k = 11 parameters, and n = 1000 model evaluations per integral, a total of 12000 model evaluations are computed.

V. Results

Uncertainty and sensitivity analyses were performed for a small (0.3 m long) mobile robot executing emergency braking and high-speed turning maneuvers. These maneuvers were chosen due to their simplicity, however more complex maneuvers such as obstacle traversal can also be studied with this approach. Figures 2 and 3 present schematics of the braking and turning maneuvers.

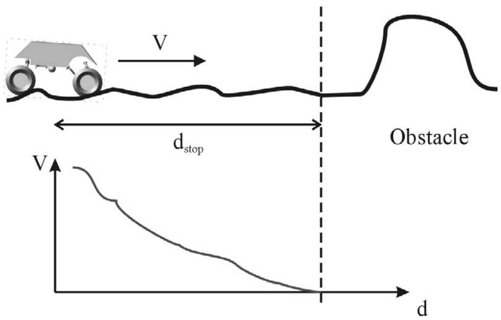

Figure 2. Schematic of the emergency braking analysis.

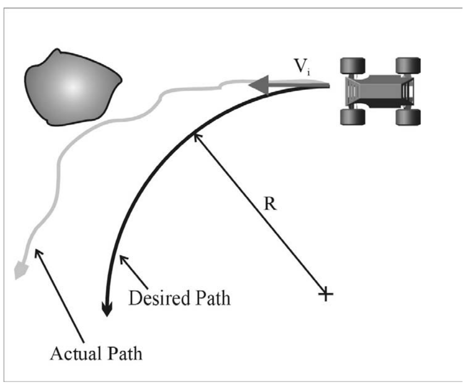

Figure 3. Schematic of the high-speed turn analysis.

The braking maneuver consists of applying a constant ("lock-up") braking torque to the rear wheels such that the robot stops in distance d_{stop}. The turning maneuver is executed by open-loop modification of the front wheels' steering angle while maintaining velocity V_i. Note that this analysis could also be applied to a system under closed-loop control. The results would then be valid for that particular control scheme.

In this analysis, the steering command results in a 90° heading change at V_i = 4 m/s. Due to model uncertainty, the stopping distance and turning trajectory of a physical system would deviate from a "nominal" model-based prediction. The deviation of the turning path from this nominal path was measured as the performance metric for the turning maneuver. The normalized root-mean-square (RMS) path error is defined as

$$E = \frac{1}{s}\sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x}_i)^2 + (y_i - \bar{y}_i)^2} \qquad (8)$$

where S is the path length, $[x_{,i}, y_{,i}]$ is the i^{th} (discretized) path coordinate, $[\bar{x}_i, \bar{y}_i]$ is the i^{th} coordinate of the nominal path, and N is the number of discrete points along the path. The RMS error is normalized by the path length to formulate an error per unit distance traveled along the turn.

The robot model presented in Section II was used for this analysis. The model parameters are presented in Table 1. To simplify this analysis it is assumed that the inertia, center of mass coordinates, and stiffness and damping properties are single parameters. The spring and damper values are assumed to be linear. The maneuvers are simulated on rough, level terrain generated using fractal techniques [19].

TABLE 1. MODEL PARAMETERS

Parameter	Name
M_v	Body Mass
I_v	Body Inertias
CG_v	Body Center of Mass
M_t	Wheel Mass
I_t	Wheel Inertias
K_t	Tire Stiffness
B_t	Tire Damping
K_v	Suspension Stiffness
B_v	Suspension Damping
μ	Ground Friction
D	Terrain Roughness

A. Uncertainty Analysis

For stopping and turning maneuvers, the input parameter space was sampled n = 1000 times at six different uniform parameter uncertainty levels, u = [0.02, 0.05, 0.1, 0.2, 0.3, 0.5]. In practice distinct uncertainty levels would likely be assigned to each model parameter; however for illustration purposes uniform parameter levels were assumed. A total of 6000 model evaluations per maneuver were computed in simulation. The initial velocity for each trial was 4 m/s.

The resulting output distribution at uncertainty level 0.1 (i.e. 10% uncertainty) is shown in Figure 4. It can be seen that a deterministic approach might predict a stopping distance of 1.6 m, whereas the stochastic approach bounds the stopping distance at approximately 1.8 m, a difference of nearly one vehicle length. This information could be used as a "safety margin" in on-line control and planning methods. In this example the resulting output distribution appears nearly Gaussian, which is reasonable considering the linear nature of the maneuver [18].

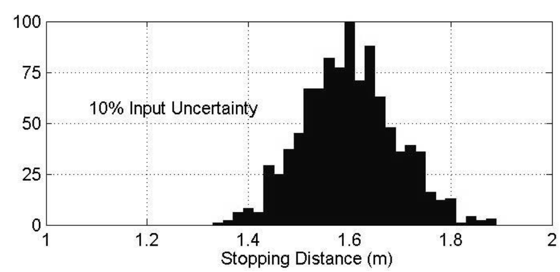

Figure 4. Stopping distance distribution for 10% uniform parameter uncertainty

A summary of stopping distance bounds (99% confidence) for each the six uniform uncertainty levels is shown in Figure 5. This shows that the bound on stopping distance increases with increasing parameter uncertainty, as expected. At high uncertainty levels, the bound grows to multiple vehicle lengths. Clearly this information would be valuable in assessing the potential safety of a given maneuver. Note that the standard deviation does not approach zero as the input uncertainty vanishes due to the stochastic nature of the fractal terrain model (i.e. different terrains can possess the same fractal dimension).

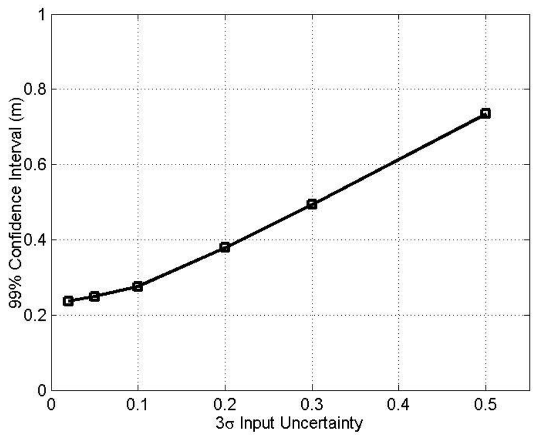

Figure 5. Stopping distance vs. parameter input uncertainty

Uncertainty in constant-speed turning was also investigated. Results from the analysis are presented in Figure 6. To illustrate the variation in turn paths, trajectories for n = 1000 samples with input uncertainty u =0.20 are plotted with the nominal path.

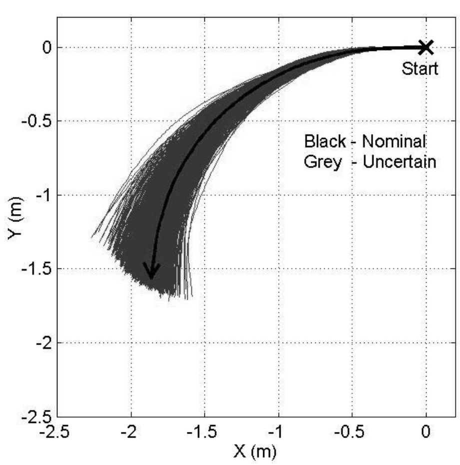

Figure 6. Uncertain turn trajectories; n = 1000, u = 0.2.

Figure 7 plots input uncertainty versus the 99% confidence interval for the turning maneuver. Again, this information could be used as a "safety margin" in on-line control and planning methods.

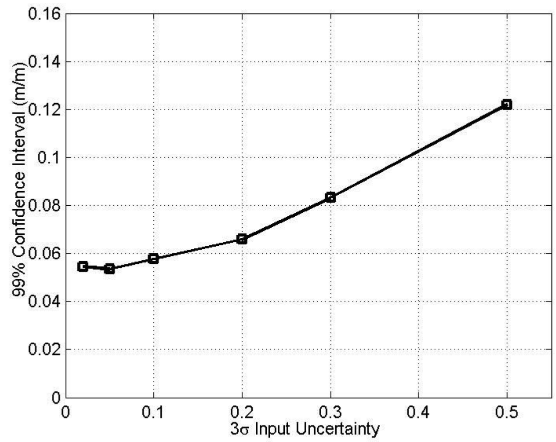

Figure 7. High-speed turning error uncertainty.

B. *Sensitivity Analysis*

Total sensitivity indices of the parameters in Table 1 were estimated for the braking and turning maneuvers. For each maneuver, the Monte Carlo sample size was chosen to be n = 1000 samples for accuracy and computational feasibility. For k = 11 parameters, a total of N = 12000 simulations were run for each analysis. The initial velocity was set to 4 m/s for both maneuvers, and the range for each parameter was uniformly taken to be 20% of the mean value. Thus we analyze the sensitivity of robot braking and turning to 20% variation in the model and terrain parameters. Again, in a real-world scenario it is likely that model parameters might be assigned unique uncertainty levels; however for illustration purposes uniform parameter levels were assumed.

The total sensitivity indices for the braking maneuver are presented in Figure 8. This indicates that braking is significantly more sensitive to tire-ground friction than the other parameters. This is expected, as the friction force generated by sliding tires is the dominant force decelerating the robot. Tire mass shows relatively high sensitivity due to ballistic motion over uneven terrain. Tires of lesser mass, for constant suspension stiffness, more closely track undulations in the terrain, while heavier tires tend to remain airborne longer. This effectively limits the friction force acting to decelerate the robot. Terrain roughness also slows the robot via energy absorption through the suspension and tires. This effect is observed in the relatively high sensitivity of the tire parameters. Tire damping is least sensitive, as variations in the damping value have negligible effects on stopping distance.

The total sensitivity indices computed for the high-speed turning maneuver are presented in Figure 9. Again, tire-ground friction is the most sensitive parameter. This result is expected, as the ground forces generated by the tire are the dominant forces acting to turn the robot. Other factors exhibit approximately equal sensitivity.

In general, this analysis indicates which parameters most strongly influence robot performance. Specifically, it suggests that terrain parameters have a significant influence on robot performance.

The difficulty inherent in estimating or sensing terrain parameters is further motivation for adopting a stochastic (rather than deterministic) modeling approach.

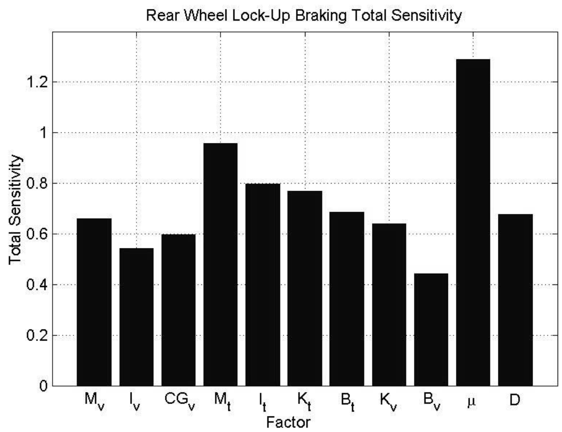

Figure 8. Braking maneuver total sensitivity indices.

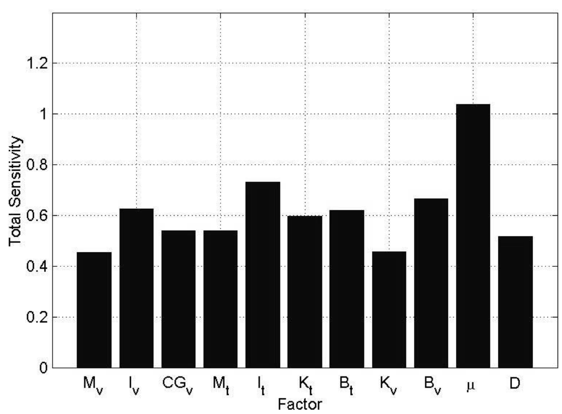

Figure 9. High-speed turning total sensitivity indices.

VI. Summary and Conclusions

This paper has presented a stochastic framework for modeling and analysis of mobile robots operating at high-speeds in rough terrain. A high-order robot model was presented. Methods for investigating uncertainty and sensitivity of robot maneuvers to input parameters were then presented. Specific examples of emergency stopping and high-speed turning were studied. It was shown that performance variation can be significant for moderate uncertainty levels. This type of analysis could provide performance bounds for predicting robot motion, which could be used in path-planning and controller development. This is an area of current research.

References

[1] J. Cunningham, J. Roberts, P. Corke, and H. Durrant-Whyte, "Automation of underground LHD and truck haulage," *Proceedings of Australian IMM Annual Conference*, 1998.

[2] M. P. Golombek, "Mars pathfinder mission and science results." *Proceedings of the 29th Lunar and Planetary Science Conference*, 1998.

[3] Y. Gonthier, and E. Papadopoulous, "On the development of a real-time simulator for an electro-hydraulic forestry machine," *Proceedings of IEEE International Conference on Robotics and Automation*, 1998.

[4] J. F. Osborn, "Applications of robotics in hazardous waste management," *Proceedings of the SME 1989 World Conference on Robotics Research*, 1989.

[5] P. J. Eicker, *The Embudito Mission: A Case Study of the Systematics of Autonomous Ground Mobile Robots*, Sandia National Laboratories, 2001.

[6] G. Gerhart, R. Goetz, and D. Gorsich, "Intelligent mobility for robotic vehicles in the army after next," *Proceedings of the SPIE Conference on Unmanned Ground Vehicle Technology*, Vol. 3693, 1999.

[7] K. Iagnemma, D. Golda, M. Spenko, and S. Dubowsky, "Experimental study of high-speed rough-terrain mobile robot models for reactive behaviors," *Proceedings of the International Symposium on Experimental Robotics*, Italy, 2002.

[8] R. M. Chalasani, "Ride performance potential of active suspension systems – part I: Comprehensive analysis based on a quarter-car model," *Proceedings of the Symposium on Simulation and Control of Ground Vehicles and Transportation Systems*, Anaheim, CA, Dec 1986.

[9] R. M. Chalasani, "Ride performance potential of active suspension systems – part II: Comprehensive analysis based on a full-car model," *Proceedings of the Symposium on Simulation and Control of Ground Vehicles and Transportation Systems*, ASME AMD, Anaheim, CA, Dec 1986.

[10] T. D. Gillespie, *Fundamentals of Vehicle Dynamics*, Society of Automotive Engineers, Warrendale, PA, 1992.

[11] S. Ikenaga, F. L. Lewis, J. Campos, and L. Davis, "Active suspension control of ground vehicles based on a full-vehicle model," *Proceedings of the American Control Conference (ACC)*, Chicago, IL, June 2000.

[12] S. J. Julier, "On the role of process models in autonomous land vehicle navigation systems," *IEEE Transactions on Robotics and Automation*, Vol. 19, No. 1, February 2003.

[13] J. C. Helton, and F. J. Davis, "Illustration of sampling-based methods for uncertainty and sensitivity analysis," *Risk Analysis*, Vol. 22, No. 3, 2002.

[14] J. C. Helton, and M. G. Marietta, "Special issue: The 1996 performance assessment for the waste isolation pilot plant," *Reliability Engineering and System Safety*, vol. 69, no. 1-3, 1996.

[15] T. E. McKone, "Uncertainty and variability in human exposures to soil contaminants through home-grown food: A monte carlo assessment," *Risk Analysis*, vol. 14, no. 4, 1994.

[16] O. Ovreberg, E. Damsleth, and H. H. Haldorsen, "Putting error bars on reservoir engineering forecasts," *Journal of Petroleum Technology*, vol. 44, no. 6, 1992.

[17] R. J. Breeding, J. C. Helton, E. D. Gorham, and F. T. Harper, "Summary description of the methods used in the probabilistic risk assessments for NUREG-1150," *Nuclear Engineering and Design*, vol. 135, no. 1, 1992.

[18] D. Golda, *Modeling and Analysis of High-Speed Mobile Robots Operating on Rough Terrain*, M.S. Thesis, Massachusetts Institute of Technology, Cambridge MA, 2003.

[19] B. B. Mandelbrot, *The Fractal Geometry of Nature*, Freeman and Company, 1977.

[20] K. Arakawa, and E. Krotkov, "Fractal surface reconstruction for modeling natural terrain," *IEEE Conference on Computer Vision and Pattern Recognition*, June 1993.

[21] I. M. Sobol, "Sensitivity analysis for non-linear mathematical models," *Math. Model. Comput. Exp.* Vol. 1, 1990.

[22] I. M. Sobol, "On the distribution of points in a cube and the approximate evaluation of integrals," *USSR Comput. Maths. Math. Phys.* Vol. 7, 1967.

[23] A. Saltelli, K. Chan, and E. M. Scott, editors, *Sensitivity Analysis*, Wiley, New York, 2000.

[24] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in C: The Art of Scientific Computing*, 2nd ed., Cambridge University Press, 1992.