A Simplified Cartesian-Computed Torque Controller for Highly Geared Systems and Its Application to an Experimental Climbing Robot

1 Introduction

There is an increasing need for mobile robots to perform tasks such as space exploration, nuclear site clean-up, bomb disposal, and infrastructure inspection and maintenance [1–3]. To function in these unstructured environments, the capabilities of multilimbed robots are desirable. However, the use of multilimbed robotic systems has been limited for a number of reasons including the fact that multilimbed systems can be difficult to control in unknown or partially known rugged environments. Effective control techniques must be developed in order to fully utilize the capabilities of these systems.

Much research has been done to develop control methods for multilimbed robotic systems in unstructured environments [4–11]. Conventional control methods such as joint PD are not suitable for such situations, since the important interactive forces or compliance with the environment can not be easily controlled. The importance of setting the stiffness in Cartesian space of a manipulated object has been shown [4]. The use of passive compliance, implemented with pneumatic actuators, was found beneficial for the control of a wall-climbing robot [5].

Many Cartesian space controllers utilize direct force feedback to control the forces applied to unknown terrain for walking in uncertain or partially known environments as in [6]. One method, called sky-hook suspension, utilizes Cartesian control of the body, but with the addition of force feedback for walking on rough terrain [7]. A more complicated hybrid position/force control scheme was applied to a walking robot in [8]. Another control scheme for walking on difficult terrain uses levels of control, including a force compliance level with force feedback [9]. However, direct force feedback of all the robot-environment forces and moments may not always be possible for a multilimbed robot. Also, incorporation of a six-axes force-torque sensor in every possible interaction point of the robot with its environment would increase the cost, complexity, and weight of the system.

Cartesian space controllers that do not require contact force sensors have been developed. One effective such controller, called Coordinated Jacobian transpose control, specifies the Cartesian stiffness between the robot’s body and ground, as well as the Cartesian stiffness between its limbs and the environment, in order to control the positions and applied forces of a multilimbed robot [10, 11]. A similar method tries to control the forces acting on the body of a biped walking robot by virtual model control [12]. These methods set a desired Cartesian impedance or Cartesian stiffness in order to control general forces and positions, without direct measurement of contact forces for feedback, by extending the traditional form of Jacobian transpose control developed for manipulators. These methods are attractive due to their ease of implementation and ability to implicitly control forces against their environment. However, they can be sensitive to unknown compliance and geometry of the environment. They are also based on the assumption that the system is static and their performance can be reduced considerably in systems that need to develop fast motions. However, this latter factor is not an important issue for most multilimbed robotic systems.

Computed torque schemes, such as operational space control, have been used in manipulators to overcome the limitations of Jacobian transpose control and improve position and implicit force control [13, 14]. These inverse dynamic schemes were applied to cooperative manipulation that used a Cartesian impedance control algorithm with increased performance over Jacobian transpose control [10, 11]. Adaptive control/walking techniques have been used to try to adapt the robot to its terrain or environment [18]. However, these methods can be difficult to implement on multilimbed mobile robots, because full dynamic models of the robot and environment are needed. Additionally, small mobile platforms with limited computational capabilities may not be able to implement these computationally expensive controllers [19].

In this paper, a simplified Cartesian computed torque (SCCT) control algorithm is developed for a highly geared climbing robot. Mobile robots that are designed to be lightweight by having highly geared transmissions have disadvantages such as high joint friction and large backlash. However, for systems with high gear ratios, it can be assumed that the actuator inertia will dominate the system dynamics. With this assumption a computed torque control can be developed that provides a computationally simple control algorithm with increased performance over Jacobian transpose control. The SCCT control scheme does not require force sensing and complex models or good knowledge of the robot and environment. The controller also allows choice of a Cartesian stiffness that permits the system to operate in partially known environ-
ments and control the interactions with the environment. The SCCT control is compared with Jacobian transpose control in theory, simulation and experimentation using the highly geared laboratory climbing robot, LIBRA, that has nonactuated end-effectors and climbs on arrays of pegs. LIBRA presents some unique multilimbed control problems, since it cannot actuate to its environment. Also, uncertainties in the pin location mandates the need for a control strategy that is robust to environmental uncertainties.

2 Cartesian Control of Manipulators

A number of Cartesian space controllers have been developed for manipulators. They have the advantage of being able to set the compliance of the endpoint in Cartesian space, so that the manipulator can be made stiff in one direction and soft in another direction. The Cartesian controllers developed for manipulators can also be applied in multilimbed robotic systems. In multilimbed robots, Cartesian controllers are needed to control the interactions of the limbs with the environment, as well as the position of the system. This could enable a robot to behave compliantly on uneven terrain in order to allow it to traverse on a difficult terrain.

There are two main Cartesian space controllers: Jacobian transpose control and computed torque control. Jacobian transpose control is a model-free controller, which uses a static transformation to transform a desired end-point force/moment into a desired joint torque \( \tau \):

\[
\tau = J^T F
\]

where \( J \) is the robot Jacobian matrix. The desired force \( F \) is calculated from a function of the Cartesian error such as in impedance control and stiffness control [19–21]. A simple Cartesian PD controller for a serial manipulator can be written as:

\[
F = K_p (X_{\text{des}} - X_{\text{act}}) + K_d (\dot{X}_{\text{des}} - \dot{X}_{\text{act}})
\]

where \( X_{\text{des}} \) and \( X_{\text{act}} \) are the desired and actual end-point position/orientation, respectively; \( \dot{X}_{\text{des}} \) and \( \dot{X}_{\text{act}} \) are the desired and actual manipulator end-point velocities, respectively; and \( K_p \) and \( K_d \) are the controller proportional and derivative gain matrices.

A limitation of the static transformation is that it does not result in a motion in the endpoint of the desired direction of the force. As shown in Section 5, this can lead to poor performance of the Jacobian transpose control in both step responses and trajectory tracking.

Cartesian computed torque control schemes such as Operational Space control [13] and resolved acceleration control [22] specify a desired acceleration at the endpoint as opposed to a force. The control scheme uses the dynamic model of the manipulator to calculate the desired joint torques \( \tau \) based on the desired acceleration \( \ddot{X}_{d} \) of the endpoint:

\[
\tau = H(\theta) J^{-1} [\ddot{X}_{d} - \dddot{\theta} \dot{\theta} + h(\theta, \dot{\theta}) + g(\theta)]
\]

where \( H(\theta) \) is the manipulator inertia matrix in joint space, \( h(\theta, \dot{\theta}) \) are the centripetal and coriolis torques and \( g(\theta) \) represents gravity, friction and other external torques.

The desired end-point acceleration \( \ddot{X}_{d} \) is calculated from a function of the Cartesian error and, as in Eq. (2), a simple Cartesian PD control has the form

\[
\ddot{X}_{d} = K_p (X_{\text{des}} - X_{\text{act}}) + K_d (\dot{X}_{\text{des}} - \dot{X}_{\text{act}}).
\]

Specifying the acceleration at the endpoint ensures that the endpoint of the manipulator moves in the desired direction, assuming the model is correct. Additionally, computed torque controllers, which use the manipulator’s dynamic models, can be tuned at higher bandwidths to improve tracking. The Jacobian transpose controller has been found, under some conditions, to exhibit unstable behaviors at higher bandwidths [14].

The increase in position tracking performance of such Cartesian computed torque schemes comes at the expense of needing to model the manipulator dynamics. Developing an accurate inverse dynamic model of a multilimbed robot operating in an unknown environment can be very difficult, and in some cases it is not feasible. Further, real-time evaluation of such models can be computationally impractical, for the small control computers carried by mobile robots. Because of the limitations of conventional Jacobian transpose control and the difficulties of implementing conventional computed torque methods, this study focused on developing an improved performance, yet practical, control method for multilimbed robotic systems.

3 Simplified Cartesian Computed Torque Control

The Cartesian computed torque control described by Eqs. (3) and (4) can be simplified in the case of highly geared robots by neglecting the centrifugal and coriolis terms and the term \( \dddot{\theta} \dot{\theta} \). For highly geared motors, the robot inertia matrix \( H \) is primarily dominated by the actuator and transmission inertia and can be considered configuration independent. Then the joint torques \( \tau \) are obtained by the equation:

\[
\tau = H(\theta)^{-1} [K_p (X_{\text{des}} - X_{\text{act}}) + K_d (\dot{X}_{\text{des}} - \dot{X}_{\text{act}})] + J^T F_{\text{ext}}
\]

where \( F_{\text{ext}} \) are external forces and moments applied at the robot endpoint. The term \( F_{\text{ext}} \) includes also the effect of the system gravity at the robot end-effector.

The block diagram of the SCCT control is shown in Fig. 1.

The SCCT control scheme is applied to a multilimbed robot, such as the robot shown in Fig. 2 as follows. The various limbs of the robot are viewed as manipulators, each doing a different task. Some limbs support and position the robot body, some carry and manipulate objects, and some apply forces to the environment. Each limb is under SCCT control and the desired reference inputs are appropriately selected for its task. For example, for free limbs that move in space, the controller shown in Figure 1 can be applied with \( X_{\text{des}} \) and \( X_{\text{act}} \) being the desired and actual position/orientation of the tip of the limb with respect to the body. For limbs that are in contact with the ground and are used to position the body, the variant of SCCT control shown in Fig. 3 can be used. In this case, \( X_{\text{des}} \) and \( X_{\text{act}} \) are the body’s desired and actual

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Fig. 1 Simplified Cartesian computed torque (SSCT) control

Fig. 2 Schematic of a multilimbed robot
position/orientation which then result in a desired acceleration for each limb with respect to the center body using the geometric transformation $T$. In computing $T$, such factors as estimates of the limb's contact compliance with the environment need to be made. Also, any closed kinematic loops formed by the robot and the environment need to be resolved [10,11]. In the following section, the effectiveness of the method as applied to the LIBRA robot is presented.

4 LIBRA: A Laboratory Climbing Robot

The laboratory climbing robot LIBRA (Limbed Intelligent Basic Robot Ascender), shown in Fig. 4, is a planar, three-legged climbing robot [10,11,23,24]. Each 32-cm limb of the 40-N robot consists of two joints driven by highly geared motors. Each gearhead has 2 deg of backlash at each joint, which can result in as much as a half-inch of error at the endpoint of each limb. Table 1 shows the values of the mechanical parameters of LIBRA limbs where $I_i$ is the inertia of the $i$th joint, $d_i$ is the viscous joint damping, and $t_f$ is the joint Coulomb friction torque. Details of the design and properties of LIBRA can be found in [23,24]. In this work, each leg on LIBRA was fitted with nonactuated hooks to allow the system to climb on pegs mounted to a wall. The Cartesian-based controller allowed LIBRA to act compliantly on the pegs through the interactions of the hooks with the pegs.

The overall experimental setup including the peg board, LIBRA, power sources and computing, is shown in Fig. 5. Seven encoder signals measure the six joints and the body’s angle with respect to the vertical. The 200 Hz control cycle updates commands to the power amplifiers which drive LIBRA.

5 Experimental Results of SCCT Control Applied to One Limb

The SCCT control scheme was implemented on one LIBRA limb, which is modeled as a two-link planar manipulator. With LIBRA in horizontal position, the effects of gravity are negligible. The inertia matrix $H$ of this highly geared leg is approximated by a 2-by-2 constant diagonal matrix whose diagonal elements are the joint inertias. Hence, neglecting the coriolis and centripetal terms, the resulting limb dynamic model takes the form

$$\begin{bmatrix}
\tau_1 \\
\tau_2 \\
\end{bmatrix} = \begin{bmatrix}
I_1 & 0 & \theta_1 & -d_1 & 0 & \theta_1 & -t_f \\
0 & I_2 & \theta_2 & 0 & d_2 & \theta_2 & -t_f \\
\end{bmatrix} \begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\end{bmatrix} \begin{bmatrix}
\tau_1 \\
\tau_2 \\
\end{bmatrix}$$

(6)

Experiments were performed to verify the basic assumption of SCCT control, that in highly geared systems the joint inertias dominate the system. Figure 6 shows a comparison of the system response predicted by the simplified model of Eq. (6) and experimentally measured response to a sinusoidal torque input to each joint. A small offset torque was added to insure a positive joint velocity. Figure 6 shows that the simple model matches the experimental system quite well.

Experiments were performed to compare the Jacobian transpose and SCCT control schemes for simple endpoint under position control of one limb. Figure 7 compares the two controllers for a step input. The tip of the limb is commanded to move 2 cm in the $Y$ direction: from point A to point B in the figure. The SCCT controller is able to hold the limb’s $X$ position essentially constant.
while moving in the Y direction. However, the Jacobian transpose control introduces a relatively large undesired motion during the commanded step. This is because the transformation contains $J^TF$ that are essentially disturbances to the system.

Figure 8 compares the two controllers tracking a circle. For slow motion (about 6 s per circle), the Jacobian transpose controller tracks the circle with some error. However, the Jacobian transpose controller error increases substantially for a fast command (about 3 s per circle). The SCCT control (with approximately the same effective stiffness) tracks the circle almost perfectly at either speed.

Figure 9 shows the greater ability of the SCCT control compared with Jacobian transpose control to produce stiff endpoint control in one Cartesian direction, while maintaining a very soft Cartesian stiffness in another. In this experiment, the tip of LIBRA’s limb is pushed away from its commanded center position of $(0.00, 0.22)$ by about 0.04 meters. The Cartesian stiffness is set very high in the Y direction; it is set to be very compliant in the X direction. Once the disturbance is removed, the limb moves back to its center position. As seen in the figure, the SCCT controller maintains the desired Y position of 0.22 m very well, while allowing the limb to be “pushed away” from its desired X position of 0.0 m. It returns to its commanded position of X equal to 0.0 with a single overshoot in the X direction. During this time Y is kept close to its commanded value of 0.00 m. The Jacobian transpose control does not perform as well. While it returns the leg to its final position, it is not able to keep the leg tip at the Y value of 0.22 m.

Figure 7 Step response of two Cartesian controllers

Fig. 7  Step response of two Cartesian controllers

6 Application of SCCT Control to Full LIBRA System

Experiments were performed to study the effectiveness of SCCT control for the full LIBRA. The results were compared with the performance obtained with Jacobian transpose control. Figure 10 shows the gait of LIBRA climbing a ladder that was selected for the results presented below. The two legs holding pegs were used to control the body position and orientation while the free leg was moving to grasp a new peg. Leg accelerations were limited in order to ensure that the legs did not pull away from the hooks at any time. Path trajectories were created for the legs and the body between discrete gait positions. A simple virtual potential was placed around the body to keep the limbs a safe distance from the body [25]. Gains for the body were experimentally determined and were made soft in order to allow the body to act compliantly on the pegs.

Figure 11 shows experimental data for LIBRA climbing two consecutive steps of a ladder task using SCCT control. Numbers illustrate the order of the actions on the plot. The climb begins with leg 1 moving from peg G to peg F. The vertical motion of the leg paths near the pins insures the hoops properly release and grasp the pins. Small errors occur at the body position as the legs pull away from hooks to grab new hooks. This is due to the selection control compliance of the body to ensure that the hooks never leave the pegs. In the experiments, the SCCT-controlled LIBRA was able to consistently climb the ladder successfully.

Figure 12 shows the compliance of the body in the X direction, while maintaining hook contact in the presence of an external disturbance. In this experiment, LIBRA is placed on the pegs in a configuration similar to Fig. 4, and an arbitrary (magnitude and direction) disturbance force applied to the body. The experiment demonstrates the ability to make the body stiffness soft in one direction and stiff in the other Cartesian directions. The compli-

Fig. 8  Experimental path tracking of the two Cartesian controllers

Fig. 9 Stiffness selection in Cartesian space

Fig. 10 One complete step in a LIBRA climbing gait

Fig. 11 Experimental bending gait
ance in the $y$ direction was set 10 times stiffer than in the $x$ direction. As expected, the motion in the $x$ direction is substantially smaller than in the $y$ direction.

As seen in Fig. 12, the position of leg #2 moved inward about 3 cm, when the Cartesian controller is initiated, from its initialized position, taking up clearance in the peg hooks and uncertainties in the peg position. Leg #1 remains at exactly the peg position $(0.457 \text{ m}, 0.152 \text{ m})$ because the body is measured in reference to that peg. As the body is pushed, both legs continue to maintain contact with the peg by constantly pulling the hook toward the peg. This ensures that the legs never leave the pegs, even when an unknown external force is exerted on the body.

Figure 13 shows a comparison experiment of Jacobian transpose control and the SCCT control. The body of LIBRA commanded to track a 5 cm in radius circle. The SCCT controller tracks the circle almost perfectly. However, the Jacobian transpose controller exhibits tracking errors for both fast tracking (about 6 s per circle) and slow tracking (about 12.5 s per circle).

A comparison of the two controllers moving LIBRA’s body upwards, following a 7.5 cm (3 in.) step vertical path in 0.5 s is shown in Fig. 14. The SCCT controller is able to maintain the desired $X$ position of 22.8 cm (9 in.) with less error than the Jacobian transpose controller.

7 Summary and Conclusions

This paper presented the development of a simplified Cartesian computed torque (SCCT) control scheme for robots with high gear ratio transmissions. The method requires a greatly simplified dynamic computation compared with conventional torque control methods. Experimental results on a multilimbed climbing robot show the validity of the assumptions and the effectiveness of the SCCT controller. The SCCT control method permits the LIBRA to climb with nonactuated end effectors due to its ability to maintain compliance with a partially known environment. The results also show that improved positioning performance is obtained by the SCCT controller over conventional Jacobian transpose control.

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