

## A Base Force/Torque Sensor Approach to Robot Manipulator Inertial Parameter Estimation

Guangjun Liu\*, Karl Iagnemma, Steven Dubowsky, Guillaume Morel\*\*  
Department of Mechanical Engineering, Massachusetts Institute of Technology  
Cambridge, MA, 02139 U.S.A.

### Abstract

*A practical method is proposed for estimating the inertial parameters of robot manipulators with substantial unmodeled joint friction and actuator dynamics. The manipulator is mounted on a six-axis force/torque sensor. Sensor measurements and joint velocities recorded during manipulator motion are used to identify the inertial parameters. The unmodeled joint friction and actuator dynamics do not degrade the estimation results, as in conventional methods. The estimation algorithm does not require difficult-to-measure acceleration measurements. Experimental results presented show that an accurate estimation of inertia parameters is attainable. Since the sensor is external to the manipulator, the same sensor can be used for parameter estimation for a number of different systems.*

### 1. Introduction

Knowledge of the inertial parameters of robot manipulators is often required for advanced control algorithms. These parameters can be estimated using the manipulator's joint torques and forces along with the joint positions and velocities [1,5,6,7,9,12]. However, most robot manipulators are not equipped with joint force/torque sensors. Thus, estimates of joint torques and forces must be used. A typical estimate is from the motor current [1,9]. A major difficulty with this method is that the joint torque/force estimation accuracy is limited by unmodeled joint friction and actuator dynamics.

A base-mounted force/torque sensor has been used to estimate mass properties of a manipulator statically [14]. The manipulator is mounted on a six-degree-of-freedom force sensor and the reaction forces and moments at its base are measured for different manipulator positions and base orientations. A procedure is developed for calculating the mass properties of a manipulator from these measurements. While this method is effective for some applications, it does not yield all the inertial properties of the manipulator. Further, it requires the reorienting of the base of the manipulator, which is usually not practical.

A base-mounted force/torque sensor was used to estimate inertial properties of a manipulator, without requiring base reorientation [15]. However, this method requires

measurement of joint acceleration, which are difficult to measure in practice.

In this paper, a method of estimating the mass, the location of the center of mass, and the moments of inertia of each rigid link of a robot manipulator during general manipulator movement is presented. The robot manipulator is mounted on an external base force/torque sensor. An estimation algorithm is derived from the Newton-Euler equations, and uses the base force sensor measurements and the manipulator joint positions and velocities. No direct measurement of the manipulator's joint torque or force are required.

A filtering algorithm eliminates the need for difficult-to-measure joint accelerations. Experimental results are presented that show that the method is accurate and effective.

The force/torque sensor measures a wrench that corresponds only to the forces and torques effectively applied to the manipulator's links [10]. Joint friction and actuator dynamics do not cause errors in the estimation results. Furthermore, as with previous methods, the base force/torque sensor is external to the manipulator. The method is cost-effective, as the same base sensor can be used with different manipulators.

### 2. Dynamic Model Formulation for Parameter Identification

Consider an  $n$ -joint manipulator mounted on a six-axis base force/torque sensor, such as shown in Figure 1. The manipulator has  $n+1$  links, where link 0 and link  $n$  are the base and the terminal link, respectively. The wrench measured by the base force sensor is denoted as  $w_s$ . The wrench at joint 1,  $w_1$ , can be obtained as

$$w_1 = T_s w_s \quad (1)$$

where  $T_s$  is a constant transformation matrix.

A local coordinate system  $P_i$  is fixed in each link  $i$  with its origin at joint  $i$ . The ten inertial parameters of link  $i$  are denoted as

$$\phi_i = [m_i, m_i c_{x_i}, m_i c_{y_i}, m_i c_{z_i}, I_{xx_i}, I_{xy_i}, I_{xz_i}, I_{yy_i}, I_{yz_i}, I_{zz_i}]^T \quad (2)$$

where  $m_i$  is the mass of link  $i$ . The coordinates,  $(c_{x_i}, c_{y_i}, c_{z_i})$  are of the center of mass of link  $i$  with respect to  $P_i$ . The elements of the inertia tensor of link  $i$  around the origin of  $P_i$  are represented by  $(I_{xx_i}, I_{xy_i}, I_{xz_i}, I_{yy_i}, I_{yz_i}, I_{zz_i})$ . It should

\* Guangjun Liu is now with AlliedSignal Canada

\*\* Guillaume Morel is now with Ecole Nationale Supérieure de Physique de Strasbourg

be noted that the inertia tensor is expressed with respect to the joint, not the center of mass of the link.

The wrench at joint 1 is related to the inertial parameters of the links as

$$w_1 = U\phi \quad (3)$$

where  $U$  is a matrix determined by kinematics and joint movement of the manipulator. The vector  $\phi$  represents inertial parameters of all links. A detailed derivation of Equation 3 can be found in [1,2].

Equations 1 and 3 yield

$$w_s = T_s^{-1} U\phi \quad (4)$$

Denoting

$$Y_s = T_s^{-1} U \quad (5)$$

gives

$$w_s = Y_s\phi \quad (6)$$

When  $N$  measurements are used, Equation 6 can be augmented as

$$W = \begin{bmatrix} w_s(1) \\ w_s(2) \\ \vdots \\ w_s(N) \end{bmatrix} \quad Y_s = \begin{bmatrix} Y_s(1) \\ Y_s(2) \\ \vdots \\ Y_s(N) \end{bmatrix} \quad (7)$$

or

$$W = Y\phi \quad (8)$$

The vector  $\phi$  can generally be estimated from Equation 8 using the least-squares method as

$$\hat{\phi} = [Y^T Y]^{-1} Y^T W \quad (9)$$

However, the least-squares method may not be applied directly when  $[Y^T Y]^{-1}$  does not exist. In this case, the ridge regression and singular value decomposition methods can be used to solve this problem [1].

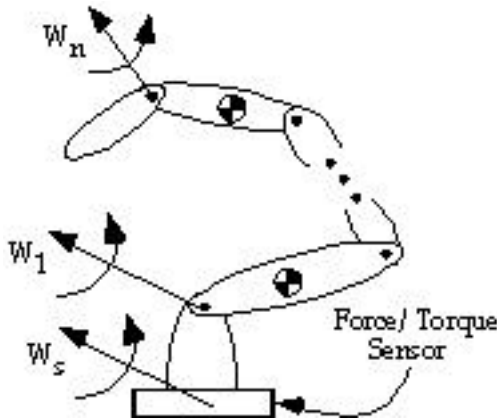


Figure 1: N-Joint Manipulator Mounted on a Force/Torque Sensor

### 3. Estimation of Inertial Parameters

#### Elimination of Acceleration Requirement

To compute the elements of  $Y$  in Equation 9, one needs the measurement of joint accelerations. However, it is difficult to measure manipulator joint accelerations directly, and it is well known that acceleration estimation using position or velocity signals is usually difficult due to noise issues. A low-pass filter transformation can help overcome this problem as studied in [5,8]. Applying a low-pass filter with unity gain at zero frequency to both sides of Equation 8 yields

$$(W)_l = (Y)_l \phi \quad (10)$$

where

$$(\cdot)_l = L^{-1} \left[ \frac{l}{l+s} L[\cdot] \right], \quad (11)$$

where  $L[\cdot]$  and  $L^{-1}[\cdot]$  represent the Laplace transformation and the inverse transformation respectively, and  $l$  is a positive constant.

Since acceleration terms in  $Y$  appear only in conjunction with functions of the joint angles  $q$ , a term containing  $\ddot{q}$  can be generally represented by  $f(q)\ddot{q}_j$  [6], and its Laplace transform is

$$L[f(q)\ddot{q}_j] = sL[f(q)\dot{q}_j] - L\left[\sum_{i=1}^n \frac{\partial f(q)}{\partial q_i} \dot{q}_i \dot{q}_j\right] \quad (12)$$

As in [6,8], applying the low-pass filter to both sides of Equation 12 leads to

$$\begin{aligned} \frac{l}{s+l} L[f(q)\ddot{q}_j] &= \\ \frac{l}{s+l} sL[f(q)\dot{q}_j] - \frac{l}{s+l} L\left[\sum_{i=1}^n \frac{\partial f(q)}{\partial q_i} \dot{q}_i \dot{q}_j\right] & \\ = l\left(1 - \frac{l}{s+l}\right)L[f(q)\dot{q}_j] - \frac{l}{s+l} L\left[\sum_{i=1}^n \frac{\partial f(q)}{\partial q_i} \dot{q}_i \dot{q}_j\right] & \end{aligned} \quad (13)$$

Applying inverse Laplace transformation to both sides of Equation 13 yields

$$\begin{aligned} (f(q)\ddot{q}_j)_l &= \\ l[f(q)\dot{q}_j - (f(q)\dot{q}_j)_l] - \left(\sum_{i=1}^n \frac{\partial f(q)}{\partial q_i} \dot{q}_i \dot{q}_j\right)_l & \end{aligned} \quad (14)$$

Thus, no acceleration term appears on the right-hand side of Equation 10.

#### Filtering Velocity Measurement Noise

Examining the first term of the right-hand side of Equation 14, we can see that it is actually the difference between the unfiltered and filtered values of  $f(q)\dot{q}_j$ , multiplied by the filter parameter  $l$ . This term is sensitive to noise contained in measurements of the joint velocity,  $\dot{q}_j$ . For example, when  $f(q)=1$ , Equation 14 becomes

$$(f(q)\ddot{q}_j)_l = (\ddot{q}_j)_l = l[\dot{q}_j - (\dot{q}_j)_l] \quad (15)$$

which could be dominated by measurement noise when the low-pass filter bandwidth parameter  $l$  is large.

To overcome this problem, the transformation defined by Equation 11 is applied again to Equation 10:

$$((W)_l)_d = ((Y)_l)_d \phi \quad (16)$$

where  $d$  is another positive constant that determines the bandwidth of the second low-pass filter.

Applying the second low-pass filter, Equation 14 becomes

$$\begin{aligned} ((f(q)\ddot{q}_j)_l)_d = \\ l[(f(q)\dot{q}_j)_d - ((f(q)\dot{q}_j)_l)_d] - \left( \sum_{i=1}^n \frac{\partial f(q)}{\partial q_i} \dot{q}_i \right) \end{aligned} \quad (17)$$

And Equation 15 becomes

$$((f(q)\ddot{q}_j)_l)_d = ((\ddot{q}_j)_l)_d = l[(\dot{q}_j)_d - ((\dot{q}_j)_l)] \quad (18)$$

Equation 16 is used to estimate  $\phi$  using the least squares technique. When  $[(Y^T)_l)_d((Y)_l)_d]^{-1}$  exists, one could estimate  $\phi$  using

$$\hat{\phi} = [((Y^T)_l)_d((Y)_l)_d]^{-1}((Y^T)_l)_d((W)_l)_d \quad (19)$$

Generally, the ridge regression or singular value decomposition methods must be used when  $[(Y^T)_l)_d((Y)_l)_d]^{-1}$  does not exist.

When implementing the parameter identification algorithm, the parameter  $d$  should be set high compared to  $l$ , but it should be low enough to filter out the velocity measurement noise. This requirement is not restrictive since the value of  $l$  could be small. This will be demonstrated in the experimental results presented in Section 4.

#### Elimination of Sensor Offset Effects

Sensor output offsets usually exist in strain gage-type force/moment sensors. Normally such offsets can be measured by taking sensor readings at zero load. However, in this method, the manipulator sits on the sensor, and the forces and moments due to gravity are mixed with the sensor offsets. This makes it difficult to measure the sensor offsets without removing the robot manipulator from the base force sensor.

In other words, the sensor output contains the motion-related wrench, gravity effects, and sensor offsets, i.e.

$$W_s = W_m + W_g + W_o \quad (20)$$

where  $w_m$  is the motion-related wrench, and it is zero when the manipulator is stationary. The gravity wrench,  $w_g$ , masks the sensor offset  $w_o$ .

The sensor offset effect can be eliminated by using only the motion-related wrench in the identification algorithm. From Equation 20:

$$W_m = W_s - (W_g + W_o) \quad (21)$$

where  $w_s$  is measured while the robot manipulator is moving along a given trajectory. Since  $w_g + w_o$  depends only on the position of the robot manipulator, it can be measured as follows: the robot manipulator is controlled to move along the same trajectory, but it is stopped at each sampling position, and the sensor output gives the corresponding  $w_g + w_o$ .

Without gravity, Equation 6 is modified as

$$w_m = Y_m f \quad (22)$$

For all  $m$  sampling points, let

$$W_m = \begin{bmatrix} w_m(1) \\ w_m(2) \\ \vdots \\ w_m(N) \end{bmatrix} \quad Y_m = \begin{bmatrix} Y_m(1) \\ Y_m(2) \\ \vdots \\ Y_m(N) \end{bmatrix} \quad (23)$$

From Equation 22:

$$W_m = Y_m f \quad (24)$$

The identification algorithm, Equation 19, can be modified using Equation 24 as

$$\hat{f} = [((Y_m^T)_l)_d((Y_m)_l)_d]^{-1}((Y_m^T)_m)_d((W_m)_l)_d \quad (25)$$

## 4. Experimental Results

### Experimental Setup

The proposed inertial parameter estimation method has been implemented and tested on a PUMA 550 robot. The manipulator was mounted on an AMTI six-axis force/torque sensor as shown in Figure 2.

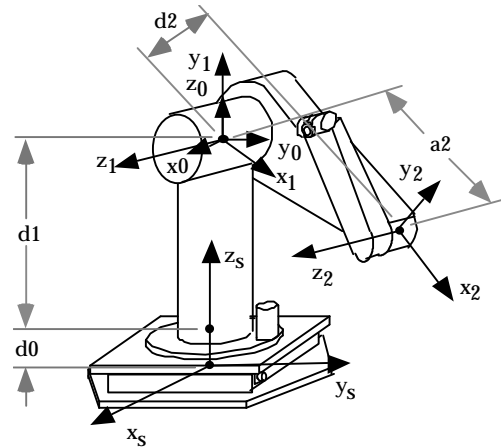


Figure 2: A PUMA 550 Manipulator Mounted on a Six-Axis Force/Torque Sensor.

In this experiment only the first two joints of the PUMA were actuated, in order to reduce model complexity. Joints three through five were immobilized in the following configuration:  $q_3=-142.1^\circ$ ,  $q_4=0^\circ$ ,  $q_5=0^\circ$ . Joint positions were measured with optical encoders, and velocity was computed by differentiating position data.

### Estimation Procedure

For the coordinate system illustrated in Figure 2, the equations relating manipulator motion to the wrench exerted at the first joint were derived as:

$$W_m = Y_m \phi$$

where

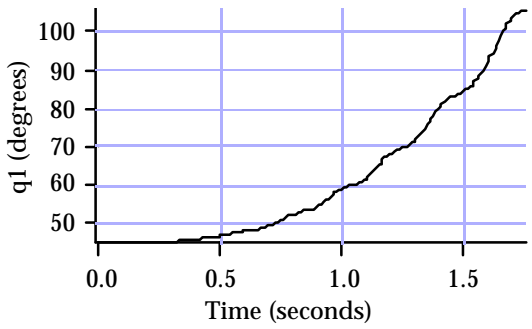
$$W_m = [F_{mx} \ F_{my} \ F_{mz} \ M_{mx} \ M_{my} \ M_{mz}]^T$$

is obtained from the base sensor measurement through a simple transformation. The vector  $\phi$  is given by

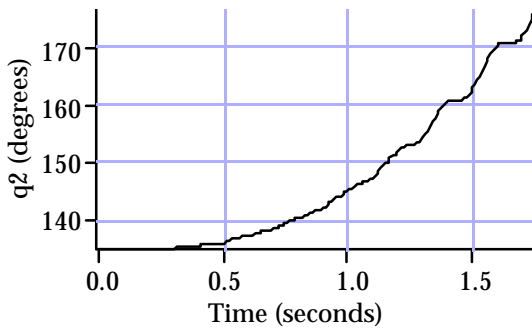
$$\begin{aligned} \mathbf{f} = & [m_1 r_{1x}, m_1 r_{1z} + d_2 m_2 + m_2 r_{2z}, \alpha m_2, m_2 r_{2y}, I_{1xy}, \\ & I_{1yy} + m_2 d_2^2 + 2d_2 m_2 r_{2z} + I_{2xx}, I_{1yz}, I_{2xy}, I_{2xz}, \\ & I_{2yy} - I_{2xx}, I_{2yz}, I_{2zz}]^T \end{aligned}$$

where  $\alpha = a_2 + r_{2x}$  and  $\beta = d_2 + r_{2z}$ . The minimum parameter set  $\phi$  is obtained analytically by combining linearly dependent columns of an original  $Y_m$  formulated using the symbolic processor Maple [4].

The excitation trajectories of the two joints are shown in Figures 3a and 3b. Joint velocities were calculated using forward-difference numerical differentiation. The sampling rate for the experiments was eight milliseconds.



(a)



(b)

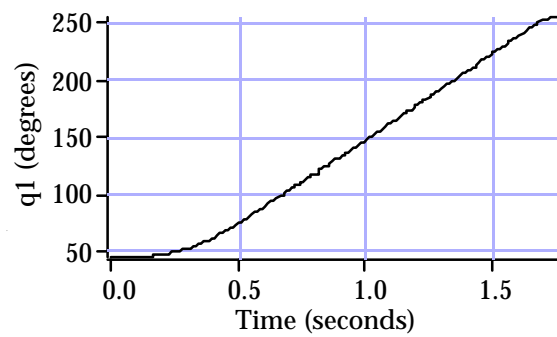
Figure 3:  $q_1$  and  $q_2$  During Identification Motion

The identification algorithm of Equation 25 was implemented in MATLAB. Filter parameters were chosen to minimize sensor noise while maintaining a good least square estimation. For the filter parameters  $l=1$ ,  $d=50$ , the following estimate of  $\phi$  was obtained:

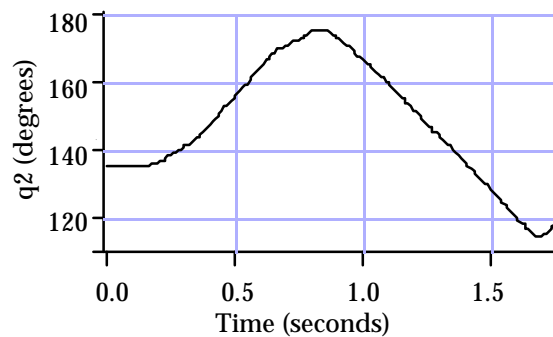
$$\hat{\phi} = \begin{bmatrix} 0.1125 \\ -4.2455 \\ 2.2202 \\ -0.1834 \\ -0.2278 \\ 1.2489 \\ 0.1338 \\ 0.1213 \\ -0.1247 \\ 0.8596 \\ -0.0086 \\ 0.9867 \end{bmatrix}$$

### Estimation Result Verification

To verify the estimation results, we used the estimated parameters to predict the forces and torques at joint 1 for a totally different motion, as shown in Figure 4. The predicted forces and torques are calculated as  $((Y_m)_1)_d \hat{\phi}$ . The filtering techniques described in Section 3 were applied. The predicted forces and torques match well those from sensor measurements as shown in Figures 5a-g.

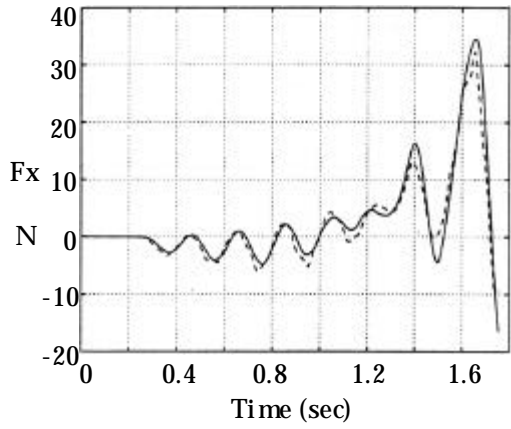


(a)

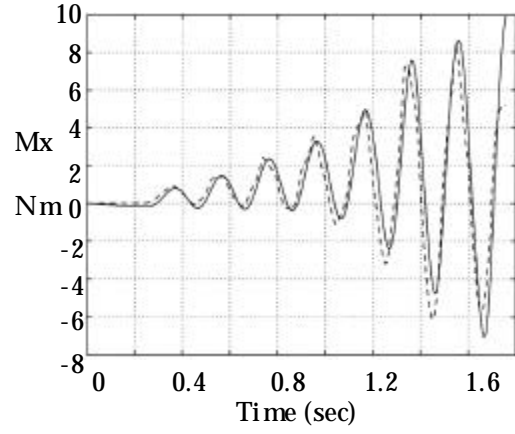


(b)

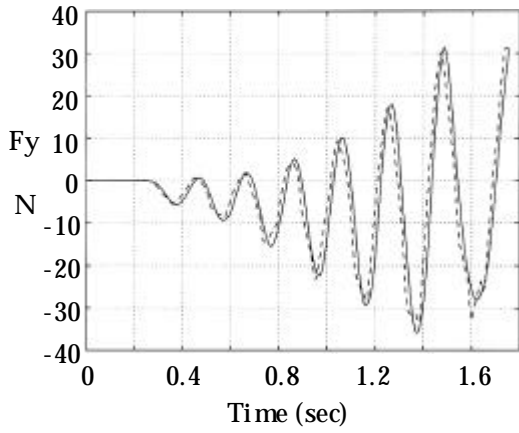
Figure 4:  $q_1$  and  $q_2$  during Verification Motion



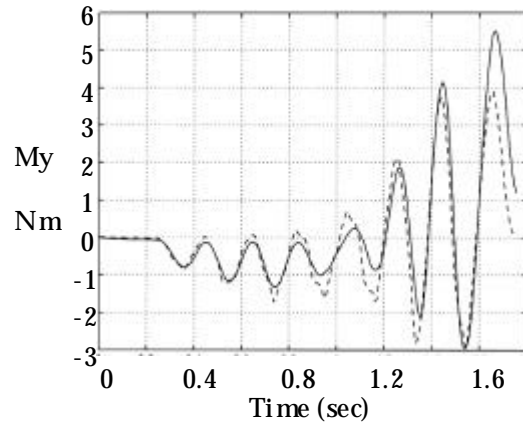
(a)



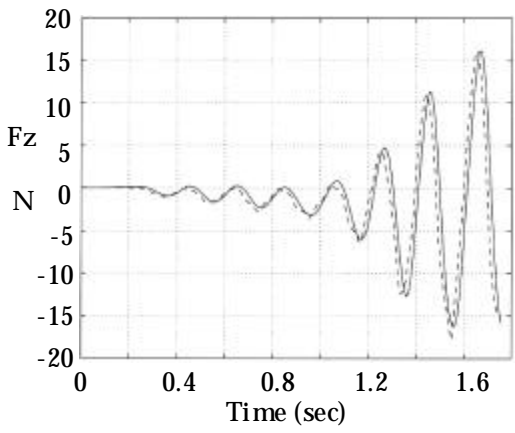
(d)



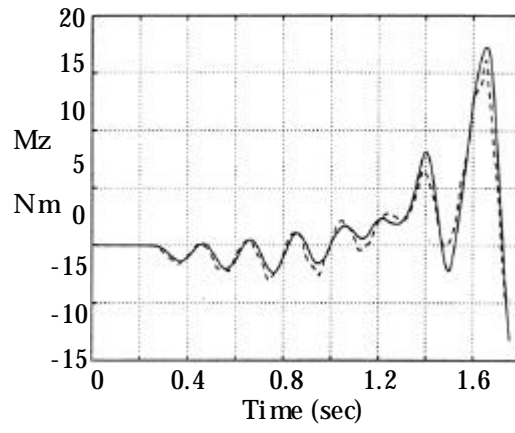
(b)



(e)



(c)



(f)

Figure 5: Predicted and Measured Forces and Torques

## 5. Conclusions

A practical method of inertial parameter estimation for robot manipulators with unmodeled joint friction and actuator dynamics is presented in this paper. The manipulator is mounted on a base force/torque sensor. The sensor measurements are used to identify the inertial parameters. The presence of unmodeled joint friction and actuator dynamics do not corrupt the estimation results. A low-pass filter technique is applied to eliminate the requirement for acceleration measurement and to reduce the effect of measurement noise of the joint velocities. The proposed method has been tested experimentally, and the results show that the estimated inertial parameters predict robot dynamics well. Since the base force/torque sensor is external to the manipulator, the same sensor can be used for parameter estimation for different robot manipulators.

The accuracy depends on the measurement accuracy of the force/moment sensor. Further work is required on systematic analysis of the estimation accuracy. Other important remaining issues are the identifiability of inertial parameters and the selection of efficient exciting trajectories. For manipulators with more than two degrees of freedom, it is nontrivial to analytically derive a base parameter set. For future work, a more general approach, similar to that reported in [4], is required.

## 6. Acknowledgements

The authors would like to thank the Korean Electric Power Research Institute (KEPRI), the Canadian Government, and the National Science Foundation for providing financial support for this work.

The authors would also like to thank Mr. Alexander Sprunt for assistance in the preparation of this paper.

## References

- [1] An, C., Atkeson, C. and Hollerbach, J., "Estimation of Inertial Parameters of Rigid Body Links of Manipulators," *Proc. of the 24th IEEE Conf. on Decision & Control*, pp. 990-995, Florida, December 1985.
- [2] Armstrong, B., Khatib, O., and Burdick, J., "The Explicit Dynamic Model and Inertial Parameters of the PUMA 560 Arm," *Proceedings of the IEEE Intl. Conf. on Robotics and Automation*, pp. 510-518, 1986.
- [3] Atkeson, C.G., An, C.H., and Hollerbach, J.M., "Rigid Body Load Identification for Manipulators," *Proc. of IEEE Conf. on Decision & Control*, pp. 996-1002, 1985.
- [4] Gautier, M. and Khalil, W., "Direct Calculation of Minimum Set of Inertial Parameters of Serial Robots," *IEEE Trans. on Robotics and Automation*, Vol. 6, No. 3, pp. 368-373, 1990.
- [5] Goldenberg, A., Apkarian, J. and Smith, H., "An Approach to Adaptive Control of Robot Manipulators Using the Computed Torque Technique," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 111, pp. 1-8, 1989.
- [6] Hsu, P., Bodson, M., Sastry, S. and Paden, B., "Adaptive Identification and Control for Manipulators Without Joint Accelerations," *Proc. of IEEE Intl. Conf. on Robotics and Automation*, pp. 1210-1215, 1987.
- [7] Khosla, P. and Kanade, T., "Parameter Identification of Robot Dynamics," *Proc. of the 24th IEEE Conference on Decision & Control*, pp. 1754-1760, December 1985.
- [8] Liu, G. and Goldenberg, A.A., "Uncertainty Decomposition-Based Robust Control of Robot Manipulators," *IEEE Trans. on Control System Technology*, Vol. 4, No. 4, pp. 384-393, July, 1996.
- [9] Lu, Z, Shimoga, K. and Goldenberg, A., "Experimental Determination of Dynamic Parameters of Robotic Arms," *Jour. of Robotic Systems*, 10(8), pp.1009-1029, 1993.
- [10] Morel, G. and Dubowsky, S., "The Precise Control of Manipulators with Joint Friction: A Base Force/Torque Sensor Method," *Proc. of IEEE International Conf. on Robotics and Automation*, Vol. 1, pp. 360-365, 1996
- [11] Popovic, M., Shimoga, K. and Goldenberg, A., "Modeling and Compensation of Friction in Direct-Drive Robotic Arms," in *Proc. of the 1993 ASME Intl. Conference on Adv. Mechatronics*, Japan, pp. 810-815, 1993.
- [12] Slotine, J.-J.E. and Li, W., "On the Adaptive Control of Robot Manipulators," *Intl. Journal of Rob. Res.*, Vol. 6, no. 3, pp. 49-59, 1987.
- [13] Spong, M.W. and Vidyasagar, M., *Robot Dynamics and Control*, New York: John Wiley, 1989.
- [14] West, H., Papadopoulos, E., Dubowsky, S., and Cheah, H., "A Method For Estimating the Mass Properties of a Manipulator by Measuring the Reaction Moments at its Base," *Proc. of IEEE Intl. Conf. on Robotics and Automation*, pp. 1510-1516, 1989.
- [15] Raucant, B., *et al.*, "Identification of the Barycentric Parameters of Robot Manipulators from External Measurements," *Automatica*, Vol. 28, No. 5, pp. 1011-1016