THE FORCE WORKSPACE: A TOOL FOR THE DESIGN AND MOTION PLANNING OF MULTI–LIMB ROBOTIC SYSTEMS

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ABSTRACT

The Force Workspace (FW) is a tool for designing mobile robotic systems which apply large forces over large ranges of motion. It can also be used to plan motions that do not violate actuator saturation limits, system-environment force and moment contact constraints such as those due to friction, and kinematic joint limits. The FW is applied to the design analysis and motion planning of a simple robotic climbing machine.

1 INTRODUCTION

Mobile multi-limb robotic systems are needed that can apply large forces while moving through large ranges of motions (Meieran, 1991, and NASA, 1989). They must not break handholds, lose footing, or overturn despite having limited actuator force and torque capacities due to weight and power consumption constraints.

A mobile, multi-limb robotic system in contact with its environment forms redundantly actuated closed kinematic chains. Hence, the actuator efforts and the contact forces and moments required for support and for the task are coupled and in general will be mathematically indeterminate. These problems have been studied extensively within the contexts of multi-fingered robotic hands and robotic walking machines. The contact forces and the nature of force-distributions, independent of contact and actuator constraints, have been studied for systems in specified configurations, (Yoshikawa and Nagai, 1991, and Kumar and Waldron, 1981). Contact forces and actuator efforts have been optimized for frictional contact constraints, (Demmel and Lafferriere, 1989) and contact forces and actuator torques

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have been determined with frictional constraints and actuator effort limits, (Kerr and Roth, 1986, Klein and Kittivatcharapong, 1990, Nahon and Angeles, 1991). Most of these studies consider systems in fixed configurations. A few have considered system motions (Klein and Kittivatcharapong, 1990, and Nahon and Angeles, 1991).

However, methods have yet to be developed to design multi–limb systems which must apply large forces over substantial ranges of motion while subject to actuator, frictional, and kinematic constraints. For example, how to design a robotic device to lift a heavy object in its “arms” while standing or climbing on a soft sandy hill, is an unsolved problem. The problem of planning the motions of such a system also remains unsolved.

These problems are considered using a method called the *Force Workspace* (FW). The FW maps system constraints into its configuration space (C-space) to form *constraint obstacles* in a similar vein to geometric C-space obstacles such as those described in (Lozano-Perez, 1987). The FW is able to represent different system constraints in a unified manner to permit rational design and motion planning, (Madhani, 1991, and Madhani and Dubowsky, 1992). First, the kinematic workspace (KW), consisting of configurations where the system motion is not constrained by kinematic parameters excluding joint range of motion limits, is established and represented in the C-space. Within the KW, additional constraints are mapped as constraint obstacles. Using the FW it is possible to determine how changes in system parameters, such as actuator torque limits and link lengths, affect a system’s ability to perform a task by observing the resulting changes in the shapes and sizes of constraint obstacles. To plan system motions which do not violate these constraints, a path in C-space is chosen which avoids these obstacles. It is shown later how a series of FW maps can be used to plan multi-step gaits for this system. The method has the limitations of C-space methods for higher dimensional problems (Caine, 1993). The FW is applied to study the design and motion plans of a simple climbing robot.

2 GENERATING THE FORCE WORKSPACE

System Description and Assumptions. The systems considered here consist of a single body from which rigid limbs extend to contact the environment or task. They may contact another kinematically constrained moving object such as a valve handle. External and task forces and moments are assumed to be known as functions of the system’s configuration. The dynamics of the system are neglected.

The system may form a mechanism with open and closed kinematic chains when in contact with the environment. If this mechanism has n degrees of freedom (DOF), then its motion is represented by the vector $\underline{q}$ with n elements q_i, where n is typically less than the number of joints. In addition, a parameter *P*, the system pose, is required to unambiguously define multiple kinematic solutions. *P* ranges from 1 to the total number of different system poses. The system C-space is parameterized by $\{\underline{q}, P\}$.

The 2^n-Tree Representation of C-space. The FW is constructed using a generalized quadtree or 2^n-tree representation of the C-space, (Samet, 1990). The nodes of the tree represent cells in the n–dimensional space. To generate the FW 2^n-tree, one or more node tests are required to determine if all configurations (C-space points) within a node are *feasible* or *infeasible* with respect to system constraints. At all configurations within a feasible node no constraints are violated, and at all configurations within an infeasible node at least one constraint is violated. If a node contains both feasible and infeasible configurations, it is *mixed*. The tree is generated by testing tree nodes, beginning with the root node, and by subdividing all mixed nodes into 2^n smaller nodes. The mixed nodes are subdivided and tested until all nodes are either feasible or infeasible, or until some pre-specified small node size is reached. In this way the problem of mapping constraints into the FW is reduced to generating appropriate and computationally practical tests which determine if a node is feasible, infeasible, or mixed.

Force Workspace Node Tests. The node tests are applied to the C-space in a hierarchical fashion in order of increasing computational complexity to build the FW. For example, the first test would generate the system KW and is relatively computationally inexpensive. The second test might map joint limits into the FW, and the third test , and most complex, maps obstacles due to actuator force / torque limits and frictional contact constraints. In this way, more complex tests need not be applied to nodes eliminated by less complex tests.

Tests for the kinematic workspaces for multi-limb systems can be difficult (Kerr and Roth, 1986). Even the tests for the joint limits of multi-limb systems are not trivial. Clearly, the development of new general and very effective kinematic tests for multi-limb mobile robots are substantial research issues and beyond the scope of this paper. This work develops a methodology, the FW, where these constraints can be applied consistently with other system constraints.

The KW node test for a particular case is given in section 3. We implement the joint limit test using a non-linear programming technique to avoid brute-force, point by point evaluation of kinematic feasibility. The joint limits are assumed in the form $p_{i,min} < p_i < p_{i,max}$, i = 1,..., # joints. For each joint, the distance between a joint position and a position half-way between the upper and lower joint limits is the objective function. This function is minimized and subsequently maximized subject to the linear constraint $\underline{q}$ node. If for these maximum and minimum values all the joint variables remain entirely within their permitted ranges over all $\underline{q}$ within a node, then the node is feasible. If any joints violate their limits over all $\underline{q}$ within a node, then the node is infeasible. Otherwise, the node is mixed.

The FW test for actuator and frictional contact constraints is developed from a well-known linear programming technique developed for robotic hands (Kerr and Roth, 1986). In the technique, the equations for static equilibrium of the system are written as:

$$\mathbf{W}\underline{c} = -\underline{F} \tag{1}$$

Each column of the 6×m matrix $\mathbf{W}$, $\underline{w}_{contact,\ i}$, is a 6×1 array of screw coordinates of the contact wrench formed from each of the components of force and moment at each contact point between the system and its environment, m is the total number of such components. The m elements of the vector $\underline{c}$ represent the scalar intensities of each contact wrench. The 6×1 wrench $\underline{F}$ represents the combination of the specified external wrenches acting on the system due to the task ($\underline{w}_{task,k}$, k = 1,...,# external wrenches). In general the system will be over constrained, where rank($\mathbf{W}$) = 6 and the null space of $\mathbf{W}$, $N(\mathbf{W})$, is not empty. In this case, the contact wrench intensities are given by:

$$\underline{c} = -\mathbf{W}^{+}\underline{F} + \mathbf{N}\underline{\lambda} \tag{2}$$

where $\mathbf{W}^{+}$ is the right generalized inverse of $\mathbf{W}$, $\mathbf{N}$ is a m×dim$N(\mathbf{W})$) matrix whose columns form a basis for $N(\mathbf{W})$, and $\underline{\lambda}$ is a dim($N(\mathbf{W})$)×1 array which may be chosen arbitrarily and which determines how the components of $N(\mathbf{W})$ will be combined. These null space components produce the “internal forces” in the over constrained system (Kerr and Roth, 1986).

The required actuator efforts for a particular system configuration are calculated as a function of $\underline{c}$ and of the task wrenches by writing the vector of actuator efforts, $\underline{\tau}$, as:

$$\underline{\tau} = J^T\underline{c} + \underline{g}(\underline{w}_{task,\,k}) \qquad (3)$$

where J is a block diagonal matrix, where the j^{th} block is the Jacobian matrix of the j^{th} limb relating differential motion of the limb tip to limb joints (Salisbury, 1982), and $\underline{g}(\underline{w}_{task,\,k})$ gives the component of the actuator efforts due to wrenches acting on the links between the corresponding contacts and each actuator.

To solve for $\underline{\lambda}$, and hence the "internal forces," unisense contact constraints (feet for example can only push when contacting objects), linearized coulomb friction constraints, and actuator effort limits are written as linear inequality constraints on the intensities of the contact wrenches, c_i. These linear inequality constraints on the c_i are mapped into a space parameterized by the elements of $\underline{\lambda}$ in equation (2) to form a constraint polygon. If the largest possible circle, or in general hypersphere, is inscribed within the constraint polygon, its center, $\underline{\lambda}^*$, will be at a maximum distance from the nearest constraint planes, and its radius, d^*, will be this distance. These are functions of system configuration and can be determined via a modification of the linear programming approach presented in (Kerr and Roth, 1986), shown in (Madhani, 1991). This technique is used to determine the feasibility of a configuration with respect to actuator effort constraints and linearized frictional contact constraints by noting that if d^* is greater than zero, a configuration will be feasible, and if d^* is less than zero, it will be infeasible.

This condition on d^* can be extended to determine the feasibility of all configurations within a C-space node cell, as required by the FW methodology. The node test begins by selecting the center point of a node and testing its feasibility using the above linear programming method. If the center point is feasible, ($d^* > 0$), a non-linear programming method is used to minimize d^* over $\underline{q}$, subject to the linear constraints: $\underline{q}$ node cell. If the resulting $d^*_{min} < 0$, the node is *mixed*, and if the resulting d^*_{min} is > 0, the node is *feasible*. If the center point of the node is infeasible, ($d^* < 0$), then instead of being minimized, d^* is maximized over $\underline{q}$, subject to the linear constraints: $\underline{q}$ node cell. If the resulting $d^*_{max} > 0$, the node is *mixed*, and if $d^*_{max} < 0$, the node is *infeasible*. This test is computationally most expensive, hence it is applied to cells within the kinematic workspace that also have been found not to violate joint limits or geometric workspace obstacles.

3 A CLIMBING MACHINE

The FW method is applied to aid in the design and motion planning of a planar, three–legged system comprising three two-link limbs joined at a single body, shown schematically in Figure 2(a). Five actuators are required, one at each "knee", and two at the body providing torque between two of the limbs relative to the third. The system climbs by using two of its three limbs at a time to pull itself upwards. The third limb is then placed at an appropriate wall location, the system's weight is transferred to it, and the procedure is repeated. (Argaez, 1993, and Sunada, et al, 1994). Figure 2(b) shows the active two limb subsystem which consists of four links and three actuated revolute joints. This sub-system pushes outwards to prevent slipping, and is limited by frictional forces at the wall and by how hard it can push given actuator torque limits. The system weight is assumed to act at the center joint. The supporting coulomb friction forces between the system's "feet" and the wall are assumed as: $||F_T|| \leq \mu F_N$, where F_N is a normal contact force, F_T is a tangential contact force, and μ is the coefficient of friction. The two limb sub-system's FW is parameterized by $\{\underline{q} = [\theta_1\ \theta_2]^T, P = 1,2\}$. The node tests for climber are discussed below.

Kinematic Workspace Test. The KW node test determines if a node lies within the KW. Note, the KW is not equal to the entire C-space for a closed chain system. The test for the climber can be found by imagining it to be separated at its body, joint 2, while each limb tip remains constrained by its respective contact location on the walls. Let $\underline{x}(\underline{q})$ denote the location of joint 2 as constrained by *limb 1*. This can easily be found as a function of θ_1 and θ_2. If these values of θ_1 and θ_2 represent a kinematically feasible position, they must lie within a region, a circle of radius (a+b) centered at the right contact, which can be reached by joint 2 as constrained by *limb 2*. Let W denote this region. Hence, a configuration parameterized by $\{\underline{q} = [\theta_1\ \theta_2]^T, P = 1,2\}$ is within the KW if $\underline{x}(\underline{q})$ lies within W.

From this constraint, a feasibility test for an entire node is developed by noting that the displacement of the tip of a serial chain will be bounded if the displacements of each of its joints are bounded, (Paden, B., et al, 1989). Then if a given joint 2 location, $\underline{x}_o$, corresponding to a given $\underline{q}_o$, lies sufficiently outside W, a test can be found which shows that all $\underline{x}$ corresponding to an entire node surrounding $\underline{q}_o$ will also lie outside W, hence the node is infeasible. Similarly, if a given $\underline{x}_o$ lies sufficiently inside W, the test will show the node is feasible. If neither case holds, the node is mixed. Following (Paden, B., et al, 1989), this test is found by bounding the displacements of $\underline{x}$ corresponding to finite displacements in the space of the parameters $\underline{q}$, or:

$$\left|\left|\frac{d\underline{x}(\underline{q})}{d\underline{q}}(\Delta\underline{q})\right|\right|_2 < B\,||\Delta\underline{q}||_\infty \tag{4}$$

where $||\Delta\underline{q}||_\infty$ is the longest side length of the node, $d\underline{x}(\underline{q})/d\underline{q}$ is a Jacobian matrix, and B is a finite bound on the displacement of $\underline{x}$, over all values of $\underline{q}$. In this case, B = a+b. The function $d(\underline{x}, W)$ which gives the minimum distance between $\underline{x}$ and W is straightforward and given in (Madhani, 1991).

Then one can perform the following test for feasibility of a particular node. If the following inequality is satisfied:

$$\frac{||\Delta\underline{q}||_\infty}{2} < \frac{d(\underline{x}, W)}{B} \tag{5}$$

then the entire node will be feasible if $\underline{q}$ at its center is feasible, or infeasible if $\underline{q}$ at its center is infeasible. If equation (5) is not satisfied, then the node is mixed. It should be noted that the above solution method for finding the work space is not directly applicable to spatial systems with closed kinematic chains and systems with both revolute and prismatic joints. The solution to this general problem remains an open research question.

Actuator Effort and Contact Wrench Constraint Test. To implement this node test, as discussed in section 2, for the climber, the following equations of static equilibrium are used. The matrix **W**, and the wrench $\underline{F}$ are:

$$\mathbf{W} = \begin{bmatrix} f_{1x} & f_{2x} & f_{3x} & f_{4x} \\ f_{1y} & f_{2y} & f_{3y} & f_{4y} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & & \\ 0 & 0 & \{\underline{r}\times\underline{f}_3\} & \{\underline{r}\times\underline{f}_4\} \\ 0 & 0 & & \end{bmatrix}, \quad \underline{F} = \left\{ \begin{matrix} 0 \\ ||\underline{W}|| \\ 0 \\ 0 \\ 0 \\ (a\cos\theta_1 + b\cos\theta_1)||\underline{W}|| \end{matrix} \right\} \tag{6}$$

where $\underline{r} = [L,h]^T$ gives the location of the right contact with respect to the origin, see Figure 2. In the particular case where both walls are vertical and parallel and L = 1, h = 0, then $\mathbf{r} = [1,0]^T$, $\mathbf{f}_1 = [1,0]^T$, $\mathbf{f}_2 = \mathbf{f}_3 = [0,1]^T$, $\mathbf{f}_4 = [-1,0]^T$, then the generalized inverse, $\mathbf{W}^+$, and the null space matrix **N** are:

$$
\mathbf{W}^{+} = \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & -1.0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 \\ -0.5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{N} = \begin{Bmatrix} 0.7071 \\ 0 \\ 0 \\ 0.7071 \end{Bmatrix} \tag{7}
$$

Dim(N(**W**)) = 1, so the space of internal forces is one dimensional. The joint torques, τ_i, are calculated as a function of the contact wrench intensities, c_i, as:

$$
\begin{Bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{Bmatrix} = \begin{bmatrix} -a\sin\theta_1 & a\cos\theta_1 & 0 & 0 \\ -(a\sin\theta_1 + b\sin\theta_2) & (a\cos\theta_1 + b\cos\theta_2) & 0 & 0 \\ 0 & 0 & a\cos\theta_4 & a\sin\theta_4 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} \tag{8}
$$

where the 3×4 matrix above is J^T and since the external loads act only on the body of the system, $g(\underline{w}_{task}) = 0$ in equation (3). These equations are used with the linear and non-linear programming methods discussed in section 2 to generate the complete actuator effort and contact wrench constraint node test, (Madhani, 1991).

The Climber Force-Workspace. The system C-space corresponds to all configurations parameterized by $\{\underline{q} = [\theta_1, \theta_2]^T, P = 1, 2\}$. Figure 3 shows the FW in $\underline{\theta}$–space for P = 1, (comprising all right limb elbow–down and left limb elbow–up and down configurations) for a system with parameters given in Table 1. A second map exists for P = 2, comprising all right limb elbow–up configurations. These parameters apply to a laboratory system discussed in Section 4 (Argaez, 1993, and Sunada, et. al., 1994).

Table 1. System parameters

Parameter	Value	Description
$\|\|\underline{W}\|\|$	37.81 N	System weight
$\tau_{1,2,3,max/min}$	± 5.74 Nm	Joint torque limits
a, b	0.152 m	Link lengths
L	0.304 m	Wall separation
h	0.0 m	Vertical Contact Separation
μ	0.8	Wall coefficient of friction.

The dark gray cells in Figure 3 are configurations outside the KW, the medium gray cells are constraint obstacles where either actuator torques or wall frictional constraints are violated, and the light gray cells are feasible configurations where the system may travel. Joint limit obstacles are not shown for clarity, see (Madhani, 1991). The white cells are formed from mixed cells. The white cells were originally tesselated to a uniform small size, however, a *merge* operation was performed on the final map resulting in the larger cells appearing in the figure.

4 THE FW AS A DESIGN TOOL FOR A CLIMBER

We give a brief example of how the FW approach can be applied to aid in the design of the climber. Such a system was designed and built in our laboratory (Argaez, 1993, and Sunada, et al, 1994). This system climbs using the "two-limb pull-ups" described earlier.

The effect of changing system parameters on the feasible FW regions, where the system can effectively support itself, can be seen in Figure 4. A more intuitive representation of the FW for this system is obtained by mapping it into X–Y space, as shown. The variables X and Y are the coordinates of the center of the body in world space. The second sub-figure shows the system superimposed on the FW. It is in a feasible configuration, since the body lies within small central feasible region of the FW. For this figure it is assumed that each joint uses the same size actuator and that for these actuators $|\tau_{i,min}|$ is equal to $|\tau_{i,max}|$. Further $\tau_{i,max}$ is nondimensionalized by WL to yield τ_{ND}. Non-dimensionalization of all design parameters results in a substantial reduction in the size of the design parameter space. The figure shows a sequence of four FW's for the system where τ_{ND} is varied from 0.411 to 0.675, and for the parameters given in Table 1. As τ_{ND} is increased, we see a steady decrease in size of the constraint obstacles. These obstacles correspond to actuator saturation and wall friction force constraints. In these figures, the feasible areas of motion for the system increases as the torque limits increase. The FW's have been used to perform "what if" trade-offs in studying the effect of varying coefficients of friction at the walls, angles of the walls, relative location of contact points, link lengths, individual actuator torques, and system weight, to give the designer a more complete picture of how the system will perform under a variety of conditions, and what changes should be made to improve performance, (Madhani, 1991).

5 MOTION PLANNING USING THE FORCE WORKSPACE

The FW can also be used to plan motions of a multi-limb mobile robotic system so that it can apply large forces and moments over large ranges of motion without violating its constraints. A series of FW's are required to plan motions requiring the relocation of the contacts. For motions with fixed environmental contact points only a single FW is required, as discussed below.

Fixed Contacts. With the FW calculated, motions between two configurations are planned by selecting a path within FW which avoids constraint obstacles. However, since cells adjacent in the FW generally do not correspond to adjacent nodes in the 2^n-tree, generating the path requires the feasible nodes of the 2^n-tree data structure to be transformed into a *search graph* whose edges represent physically adjacent cells (Madhani, 1991, and Samet, 1982). Then a path between two connected points is found using a minimum cost graph search, (Sedgewick, 1990). To tailor a path for a given application, the edges of the search graph are weighted with a configuration based performance index, and the path with the minimum overall path weight is selected.

Two examples for the climber, using the parameters of Table 2, are discussed briefly below. First, the edges of the search graph were weighted by the distance between nodes. In the resulting path the center body travels the minimum distance between two points, see Figure 5. While this path avoids the constraint obstacle in the right of the FW, the actuator torques for joint 3 remain very close to saturation for a substantial part of the motion, see Figure 6. For this path the left foot is also on the verge of slipping for a part of the path (Madhani, 1991).

Table 2. System parameters

Parameter	Value	Description
$\underline{\|\|W\|\|}$	300 N	System weight
$\tau_{1,2,3,max/min}$	± 200 Nm	Joint torque limits
a, b	0.5 m	Link lengths
L	1.0 m	Wall separation
h	0.2 m	Vertical Contact Separation
μ	0.5	Wall coefficient of friction.

To prevent this behavior a different search graph edge weighting function can be applied. Recall that when a system is far from its actuator and friction constraints,

d^*, the minimum distance from the nearest linearized actuator/friction constraints in the space of internal forces, will tend to be large. Hence we apply the following edge weight to the search graph:

$$\text{edgeweight} = \| \underline{x}_1 - \underline{x}_2 \|_2 \left[\left((1/d^*_1) + (1/d^*_1) \right) / 2 \right]^{\alpha} \qquad (11)$$

where d^*_1 and d^*_2 are the values of d^* calculated at the centers of adjacent nodes which define an edge on the search graph, and $\underline{x}_1$ and $\underline{x}_2$ are the (x,y) coordinates of the body at the centers of these nodes. The resulting path minimizes an approximate integral of $(1/d^*_1)^{\alpha}$ along the path. Setting α equal to 5 strongly penalizes paths which enter areas of low d^*.

Figure 5 shows the resulting path. Figures 7 and 8 show the corresponding actuator torques and the ratio of tangential to normal contact forces respectively. While the d^*-based criteria steers clear of infeasible regions, the body traverses more than twice the distance of the minimum distance path. Figure 7 shows that the joint 3 torque remains away from the it saturation limits. Figure 8 shows the ratio of tangential to normal contact forces remain away from the ratio of 0.50 (corresponding to the assumed coefficient of friction) and hence slipping is not a critical issue until the end of the path, which is near an infeasible region, and hence cannot be avoided.

Planning a "Gait." The above procedure allows motions to be planned for a system given that a fixed set of contact conditions exists between the system and its environment and task. During many tasks, however, it may be desirable to change or relocate contacts, for example while making a step with a walking machine or while turning a valve "hand-over-hand". We use the term *stance* to define a system and a particular set of contacts with its environment and task; a system in a given stance forms a particular mechanism and has associated with it a single FW in which motions can be planned using the above computer search technique. Each time a new set of contacts is chosen, the system forms a new mechanism, or *resides in a new stance*, and a new FW will be generated to plan motions for this mechanism. During the process of transferring between stances, or equivalently between FW's, we must ensure that system constraints are not violated.

This can be done even though the particular system variables used to parameterize the system DOF will in general differ between stances. In order to change stances, the topology of the mechanism formed by the system and the environment must change. For example, a limb may be brought into contact with the environment or the system's task, and subsequently, another may be lifted to complete the transfer between stances. Any external loads or task forces must be supported throughout this process. The necessary and sufficient condition to allow the accompanying transfer of forces between limbs during transfer between stances is that the system configurations before and after the shift must lie in feasible regions in each of the corresponding FW's, (Madhani, 1991). The system may have to move from its current configuration in order to be compatible with a feasible configuration in the next stance. After the transfer, a new set of q_i's will in general be required to parameterize the motion of the new mechanism formed by the system. Continuing, the result can be referred to as a *gait* for the multi-limb system, in the sense of a walking machine, although it applies to any multi-limb system moving through a series of stances.

Figure 9 shows a FW produced gait for the climber system, with parameters given in Table 2 modified with $\tau_{1,2,3,max/min}$ = 300 Nm, μ = 0.8, and h = 0.4 m. The light gray regions represent feasible system configurations, and the dark gray region in Figure 9 (b) is the feasible intersection of the FW's corresponding to each stance. Since such a feasible intersection exists, it is possible to choose a new contact point that is feasible. The condition on the system configuration for the foot force transfer to occur is that the body lie within this intersection. Once contact forces are shifted to stance 2, the unloaded limb can be lifted and planning can continue within the new FW. Continuing cyclically in this manner produces the final gait with the resulting body motion as shown in the figure.

6 SUMMARY AND CONCLUSIONS

The Force Workspace method has been presented to aid in the design and motion planning of a class of mobile, multi-limb systems which must apply or support specified loads over large ranges of motion without violating actuator force / torque saturation limits, contact force / moment constraints between such systems and the environment (friction), and constraints on joint ranges of motion. The method uses a recursive subdivision process to map the above constraints in a unified way into the system C-space to form *constraint obstacles*. The effects of changes in specific design parameters can be observed through changes in the sizes and shapes of the constraint obstacles. Motions can be generated which do not violate system constraints by selecting paths that avoid constraint obstacles. These paths can be optimized based on a configuration dependent performance index. "Gaits" for a multi-limb system can be generated using the approach if a task requires that limbs change the type or location of contacts with the environment. Extensions exist to consider power consumption, (Dubowsky, Moore, and Sunada, 1995).

In theory, the algorithms and analyses used to generate the FW and for planning motions within it can be applied to higher DOF systems, however its practical extension to higher order systems raises some important questions. First, as with any C-space method, direct graphical visualization for systems with more than three degrees of freedom is not possible. Methods have been developed to visualize 3D C-space (Caine, 1993). Nevertheless, the FW generation and planning algorithms developed apply to higher order C-space despite lack of direct visualization. A possibly more important issue is the increase of computational complexity for high degree of freedom systems. The major issue is the growth in the number of cells as the dimensionality of the space increases. Other increases in computational burden, such as applying linear and nonlinear programming techniques in higher order space appear less critical. Applying evaluation tests in a hierarchy based on computational effort and continually varying the tessellation size of the space should be relatively efficient compared to other methods. Simple analysis suggests that applying the methods presented in this paper to a mobile system in contact with the environment with more than four to six degrees of freedom would tax present day computers, (Madhani, 1991).

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