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Chaotic Vibration and Design Criteria for Machine Systems with Clearance Connections

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Abstract: *The dynamic responses of some machine systems with clearance connections and component flexibility are very sensitive to even small variations of system parameters. This hypersensitivity is associated with potential system chaotic behavior. It is shown that it can limit the usefulness of the predictions of computer simulations for design. Analytical and experimental results suggest the dynamic responses of these systems can be classified into three characteristic types. It is suggested that these characteristics must be considered in the dynamic analysis used for the design of high performance machine systems.*

Keywords*:* Machine dynamics, chaotic vibrations, design analysis, hypersensitive response, dynamic modeling

Introduction

Joint clearances and component flexibility in a machine system can substantially degrade its performance, causing impacts, vibration and noise, component fatigue, and poor precision. Many analytical design models of the dynamic effects of clearance connections and component flexibility have been developed. Over the past three decades, a number of methods have been proposed to model the dynamic response of machines with clearance connections and component flexibility [1-6]. More recently, studies have revealed the dynamic responses of these systems are quite complex [7-12]. These studies have showed analytically and experimentally that simple systems with clearances and impacts can exhibit very complex dynamic behavior, such as subharmonic and chaotic vibrations. They also suggested that more complete machine systems, some with multiple clearance connections and nonlinear kinematic motions also exhibit complex behavior. By and large, these studies have focused on basic mechanics issues, such as demonstrating the existence of classical chaotic behavior. They have not addressed the implications for the design of such complex behavior.

Recently, it has been noted that the dynamic responses of machine systems with clearance connections seemed to exhibit both a large variation and hypersensitivity to small variations of system parameters, even for periodic system response [7,13].

This paper reports on an analytical and experimental study on the design implications of chaotic behavior in machine systems with clearance connections and component flexibility. The results of this study provide some fundamental understandings of the characteristics of machine systems with clearance connections and component flexibility, and important guidelines for design. It is shown that in addition to simple periodic response, called here Type I Response, and chaotic response, called Type II Response, under certain conditions, a machine system that can exhibit chaotic vibrations, will also exhibit hypersensitivity in its periodic behavior to design parameters. This behavior is called Type II response. In Type II, the dynamic behavior of a real machine system might have be quite from the response predicted by a "valid" analysis, because the machine, when manufactured and used, is subject to variations in its parameters, such as its component dimensions and material properties. This Type II character could represent an important limitation to computer-based dynamic analysis for design. These conclusions are based on the study of two systems: a simple experimental system called an Impact Beam System (IBS), and a more realistic system called a spatial slider crank (SSC). If further research confirms this result for a wider range of systems then this characteristic must be considered when dynamic analysis is used in support of the design of high performance machine systems. A detailed discussion of the spatial slider crank results is beyond the scope of this brief paper.

The dynamic modeling method

The dynamic modeling method used in this study is based on a technique developed for flexible, spatial machine systems with clearance joints [6,15]. It uses Hartenberg-Denavit 4x4 transformation matrices to represent the nominal motion of the system components, called links. The distributed mass and flexibility of the links are modeled using finite elements. The motions of the FE nodes with respect to the nominal motion frame of each link are described using perturbation coordinates. The dynamic equations of motion for each link are written using Lagrange's formulation in which perturbation coordinates are generalized coordinates. These equations are reduced by Component Mode Synthesis (CMS) to improve computational efficiency but maintain certain dynamic accuracy. Compatibility matrices, which express the kinematic or force constraint relationships between the links, are used to assemble the system dynamic equations of motion in the form [6, 15]:

$$\bar{\mathbf{M}}\ddot{\mathbf{q}} + \bar{\mathbf{G}}\dot{\mathbf{q}} + \bar{\mathbf{K}}\mathbf{q} = \mathbf{Q} \tag{1}$$

The time varying matrices $\bar{\mathbf{M}}$, $\bar{\mathbf{G}}$, and $\bar{\mathbf{K}}$ in Equation (1) describe the mass, damping and stiffness characteristics of the system. The $\mathbf{q}$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are the global independent displacement, velocity and acceleration vectors. The generalized force vector $\mathbf{Q}$ accounts for actuator force/torques and external loads, etc.

The matrices $\bar{\mathbf{M}}$, $\bar{\mathbf{G}}$, and $\bar{\mathbf{K}}$ are functions of the link element mass, stiffness matrices, the link CMS transformation matrix $\mathbf{A}_i$, the link compatibility matrices $\mathbf{B}_i$, and link joint nominal motion variables θ_i. They are defined as follows [6] :

$$\bar{\mathbf{M}} = \sum_{i=1}^{NL} \mathbf{B}_i^T \mathbf{A}_i^T \mathbf{M}_i \mathbf{A}_i \mathbf{B}_i \,, \tag{2}$$

$$\bar{\mathbf{K}} = \sum_{i=1}^{NL} \mathbf{B}_i^T \mathbf{A}_i^T \mathbf{K}_i \mathbf{A}_i \mathbf{B}_i + \sum_{i=1}^{NL}\sum_{j=1}^{NL} \mathbf{B}_i^T \mathbf{A}_i^T \mathbf{G}_i \mathbf{A}_i \mathbf{B}_{ij}\,\dot{\theta}_j + \sum_{i=1}^{NL}\sum_{j=1}^{NL} \mathbf{B}_i^T \mathbf{A}_i^T \mathbf{M}_i \mathbf{A}_i \left[\sum_{k=1}^{NL} \mathbf{B}_{ijk}\,\dot{\theta}_j\,\dot{\theta}_K + \mathbf{B}_{ij}\,\ddot{\theta}_j\right] \tag{3}$$

$$\bar{\mathbf{G}} = \sum_{i=1}^{NL} \left\{ 2\mathbf{B}_i^T \mathbf{A}_i^T \mathbf{M}_i \sum_{j=1}^{NL} \mathbf{B}_{ij}\,\dot{\theta}_j + \mathbf{B}_i^T \mathbf{A}_i^T \mathbf{G}_i \mathbf{A}_i \mathbf{B}_i \right\} \tag{4}$$

$$\mathbf{Q} = \sum_{i=1}^{NL} \mathbf{B}_i^T \mathbf{f}_i \,, \tag{5}$$

NL is the number of links in the system.

$\mathbf{B}_{ij}$ is $\frac{\partial \mathrm{B_i}}{\partial \theta_\mathrm{j}}$, and $\mathbf{B}_{ijk}$ is $\frac{\partial^2 \mathrm{B_i}}{\partial \theta_\mathrm{j} \partial \theta_\mathrm{k}}$.

The $\mathbf{B}_i$'s, the compatibility matrices, used to construct the equations of motion of the multibody system, are determined by the nature of the system's joints. These joints may be ideal, meaning that they apply kinematic constraints to the system, or they may be non-ideal with compliance and clearances.

A numerical simulation package, called ASSET, implements this technique[13]. ASSET was used to study the Impact Beam System and Spatial Slider Crank as discussed in the following sections.

The Impact Beam System (IBS)

The system and its model. The IBS was developed to study machine systems with clearance connections and component flexibility [13,14], see Figure 1. The IBS that consists of a beam, a one-dimensional clearance and a base plate. Each element of the IBS represents a feature of typical machine systems: the beam (steel) represents a flexible component; the clearance connection represents a bearing with internal clearance; the base plate represents a supporting structure. One end of the beam is held by a flexure pivot, providing a zero clearance bearing. The other end of the beam is inside the one-dimensional clearance joint with adjustable clearance. An electrodynamics shaker applies a controlled periodic force to the beam so impacts are generated in the clearance joint when the beam tip reaches the clearance. The adjustable clearance joint is instrumented with piezoelectric contact force sensors to measure directly impact forces at their tips. Clearance used in this study ranges from 0 mm (no clearance) to ±0.25 mm which is a typical range of the clearance found in many machine systems.

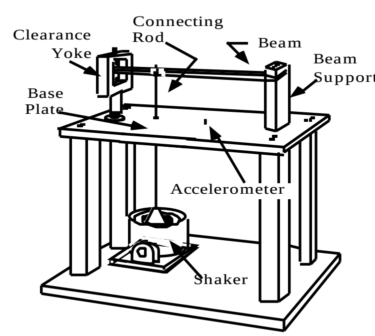


Fig 1. The Experimental Impact Beam System

The beam is modeled using finite elements. The flexure pivot is modeled as two springs. One spring is in one rotational degree of freedom and the other is in one translation degree of freedom. The remainders of the system are treated as rigid. The model of the system is shown in Figure 2. The clearance connection is modeled as a zero force zone to represent the gap and a non-zero force zone with linear contact stiffness and damping to represent the surface compliance of the connection. The stiffness is calculated using a linearized Hertzian contact analysis. The contact stiffness and damping used for the experimental setup are 1.5 $\times 10^7$ N/m and 20 N-s/m, respectively [13]. The shaker input is modeled as a force applied to a beam FE node corresponding to the shaker attachment point. The shaker's suspension is modeled by a linear stiffness and a damper as Ks and Cs.

The IBS dynamic response. The model of IBS was simulated using ASSET. The results predicted the existence of chaotic behavior at certain excitation frequencies and clearances, and to be periodic in other cases. In Figure 2, the phase plane portrait of the response of the beam tip inside the clearance joint is presented for ±0.127 mm clearance and 19 Hz excitation frequency. The close orbit, response is clearly periodic.

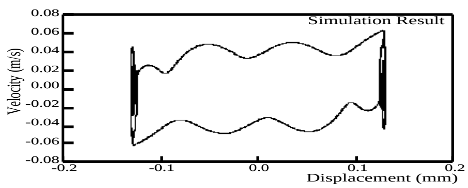


Fig 2. A Periodic IBS phase plane portrait.

Figure 3 shows the phase plane portrait of the beam tip motion inside the clearance joint for ±0.127 mm clearance and 30 Hz excitation frequency. This motion does not repeat from cycle to cycle. It is chaotic.

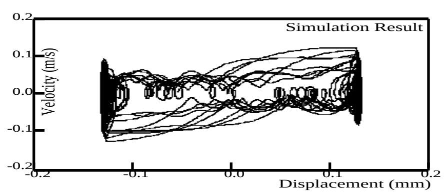


Fig 3. A chaotic IBS phase plane portrait.

This is confirmed by the Poincaré map for the response shown in Figure 4. Simulations were used to determine parameter ranges that resulted in chaotic vibrations. The amplitudes of impact forces are important for machine design, having a strong influence on machine's fatigue life, vibration and noise, etc. The impact force on one side of the bearing for this case is shown in Figure 5.

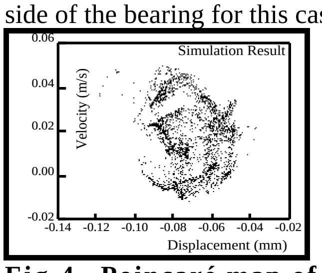


Fig 4 Poincaré map of IBS chaotic motion.

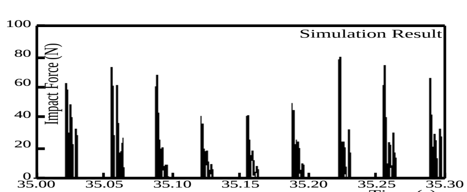


Fig 5 Impact force for chaotic motion.

Where the response was chaotic the impact force varies very substantially from cycle to cycle, as shown in Figure 5. Variations in the peak impact force of factors of two or three were common.

Extensive experiments verified the validity of the IBS simulations [13]. Figure 6 shows the experimentally measured impact force on one side of the bearing for ±0.127 mm clearance and 30 Hz excitation frequency. This figure shows large variation of the impact force seen in the simulations of the chaotic cases. The experimental and simulation studies showed that a number of factors, including clearance size, excitation frequency, and beam dimension affect the character of the system response.

Figures 7 shows peak impact forces as functions of the excitation frequency for ±0.127 mm clearance. This figure shows the impact forces for 100 cycles of IBS operation. For a periodic response, a single point appears since the peak values of 100 cycles are the same. For a subharmonic response, a few discrete points appear. For a chaotic response, distributed points appear since the peak force varies from one excitation cycle to another. The responses can be classified into three types based on the characteristics of their impact force, such as shown in Figure 7. Type I Response is periodic, and is not sensitive to initial conditions or small variations of system parameters. The predicted impact force increases smoothly as the excitation frequency increases. This well-behaved region permits the designer to predict trends accurately or without ambiguity.

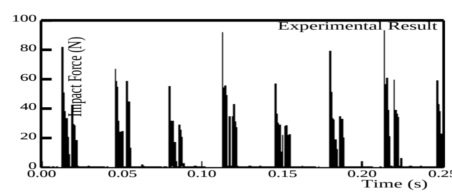
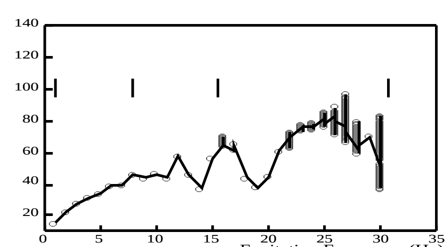


Fig 6. Measured chaotic motion impact force.



tion frequency changes only small amounts. For the Type II Response, the hidden danger is its periodicity. The periodic response may lead designers to overlook its sensitivity to small variations of system parameters. Type III Response is unpredictable, either chaotic or periodic, as shown in Figure 7 for frequencies between 16 and 30 Hz. When chaotic, the predicted impact force is sensitive to initial conditions. For example, at 30 Hz, the peak impact force varies between 30 N and 85 N, depending on the initial displacements and velocities. The peak impact force also fluctuates significantly as the excitation frequency changes. For example, while the response is periodic with a peak impact force of 70 N at 29 Hz, it becomes chaotic with peak impact force in the range of 30 - 85 N as the excitation frequency increases to 30 Hz. For the Type III Response, the uncertainty and sensitivity of the responses present ambiguities for a designer or analyst using the predictions of computer-based simulations.

These three types of behavior are also evident in the experimental results. Figure 8 shows the experimental results that are comparable to the Figure 7 case. The form of the experimental results are quite similar to the simulation results. One would not expect precise agreement, as small differences between the parameters used in the simulation to represent the experimental system will result in large differences of behavior in the Type II and Type III regions by their very nature. Due to small measurement uncertainties, the periodic responses (both Type I and Type II) in this figure exhibit small variations.

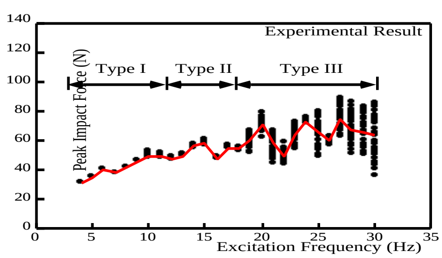


Fig 8. Experimentally measured peak impact force as a function of frequency.

Using both the simulations and experimental systems the response of the system was mapped as a function of dimensionless clearance and excitation frequency. Dimensionless clearance is the clearance normalized by the displacement of the beam tip if a static force, equal to peak magnitude of the excitation force, were applied to the unconstrained beam. In the figure, an open circle represents a periodic response, and a black dot represents a chaotic response. Based on the characteristics of the responses in this parameter space, the corresponding regions for these three types of responses are suggested in Figure 9 using three different shadings. This figure suggests that large clearance and high frequencies tend to lead to a chaotic response.

The difficulty of designing systems that exhibit Type II or Type III behavior is illustrated by Figure 10. It shows the peak impact force as a function of the variation of the beam length for 29 Hz excitation frequency and ±0.127 mm clearance. The design length of the beam is changed increments that are about 1% of its nominal length. These small variations change the response from chaotic to periodic or vice versa. When the beam length increases by about 1% from 281.5 to 284 mm, the response changes from periodic to chaotic, and the maximum value of the peak impact force increases from 70 N to 95 N, almost a 30% increase. For such a system the dynamic behavior of a machine could be quite different from design predictions. These differences are due to small dimensional variations, uncertainties in the values chosen for model parameters, inconsistencies between assumed and actual boundary conditions, operating condition variations, and other such effects.

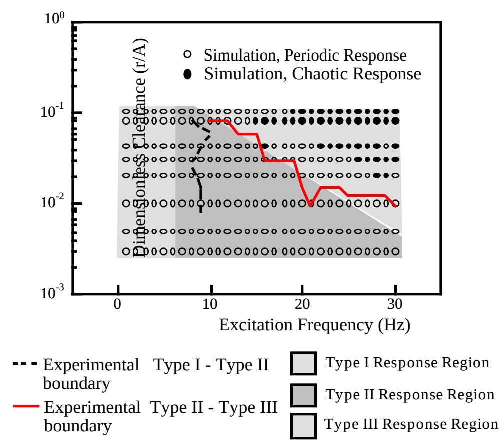


Fig 9. Measured and simulated Type I, II and III regions for IBS system.

The study of a spatial slider crank

In this study an instrumented spatial slider crank was designed and tested. This work showed that many of the characteristic behaviors found for the IBS were evident in the slider crank, including the existence of Type I, II and III behaviors. Suggesting the relevance to results presented above for the IBS systems for more complex and realistic machine systems. A complete report of this phase of the study is beyond the scope of this brief paper. Reference [14] contains a detailed description of this phase of the work.

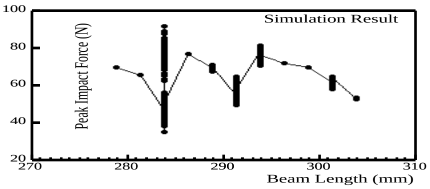


Figure 10. Impact force versus beam length.

Conclusions

The results presented in this paper show that the complex nonlinear behavior found in systems with clearance connections can make the design of even simple systems complex. Chaos (Type III behavior) and its accompanying Type II behavior can result in substantial increase of the important peak impact forces, that cause vibration and noise, wear, and shorten system life. Due to the variation of impact force from cycle to cycle, in a chaotic response, it is difficult to determine maximum impact force from just a few simulation cycles. Even though the computation time required to simulate a complex nonlinear machine with flexible component and clearance can be very large, design analysts must perform a number of simulations run for sufficient time to insure that they properly characterize the system's behavior. In our work we were not able to find a quantitative measure of the number of cycles required to characterize a given system with a high degree of confidence.

It is also clear from the results presented that chaotic and Type II response, with their increased impact force, can lie in the neighborhood of well-behaved responses. A machine designed by studying its response for only its nominal design parameters machine may fail, when manufactured or after time in the field, when small variations in system parameters or operating conditions result in very substantial changes in system performance. The key result obtained in this study is that for systems of this class special care must be taken in their design when dynamic performance is important. The designer cannot simply perform dynamic simulations for a few machine cycles at a few design points for a system with clearance connections and component flexibility and assume that the system behavior is well characterized.

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