DISPLACEMENTS IN A VIBRATING BODY BY STRAIN GAUGE MEASUREMENTS

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ABSTRACT

An important issue in the position control of elastic systems is the correct on-line measurement of displacements. End-Point Control of flexible manipulators, for example, requires the measurement of the manipulator's tip displacement. While this kind of measurement is relatively easy to carry out in a laboratory setting, it can be problematic in a real world environment.

A novel procedure for the real-time determination of displacements at any point of a vibrating body is proposed. The procedure calls for the use of strain gauges and the knowledge of the modal properties of the structure. Advantages of the proposed method are its simplicity and low cost.

To verify the method a simple test structure (a clamped-end beam) was instrumented and experimented upon under different loading conditions. The displacements derived with the proposed method show good agreement with displacement measurements performed with a Laser Doppler Vibrometer.

NOMENCLATURE

- $\mathbf{r}$ position vector
- t time variable
- m mass per unit volume/area/length
- c damping coefficient per unit volume/area/length
- $\mathbf{u}$ displacement vector
- $\mathbf{x}$ vector of coordinates
- L linear differential operator in the spatial variables
- x_i coordinate along reference axis i = 1, 2, 3
- E Young's modulus
- D plate stiffness
- I second moment of inertia of beam's cross section about its neutral axis
- h plate thickness
- ν Poisson's ratio
- Ψ_j j-th normal mode of the system for undamped free vibration
- V volume/area/length
- ω_j j-th undamped natural frequency
- δ_{ij} Kronecker operator
- y_j modal coordinate for mode j
- γ_j modal bandwidth for mode j
- Φ_j strain eigenvector tensor for mode j
- y_j^ε strain modal coordinate for mode j
- ε strain tensor
- ϕ_{jkl} (k, l) component of strain eigenvector tensor for mode j
- n number of modes taken into consideration
- $\mathbf{r}_m$ vector defining the position of strain gauge m
- $\bar{\mathbf{r}}$ vector defining the position of the point whose displacement is to be determined
- $\mathbf{e}$ unit vector defining the direction of the displacement to be determined
- U Fourier Transform of the displacement signal
- E Fourier Transform of the strain signal
- Y Fourier Transform of the modal coordinate
- U/E transfer function between the displacement and the strain signal

1. INTRODUCTION

Real-time displacement measurement in vibrating bodies is a very important issue in applications related to vibration reduction in elastic structures [1] or to position control of flexible manipulators [2]. End-Point Control, for instance, requires on-line measurement of the manipulator's effective tip position to be used in the feedback law. While there are many transducers that can measure displacements, their performance in this type of application is never fully satisfactory, either due to range limitations of the transducers themselves, especially with respect to the range of motion of a typical manipulator, see for example [2] and [3], or due to the complexity of the measuring apparatus, as in the case of [4].

In order to overcome these problems other type of transducers could be used. Piezoelectric accelerometers provide displacement information by double integration of the acceleration signal, but in addition to adding mass to the system, their performance is not satisfactory at low

*Currently at ENERECO S.C.a.r.l., via Canale Albani, 13, 61032 Fano (PS), Italy frequencies. Velocity transducers on the other hand suffer from the same drawbacks as displacement transducers in this type of application.

This paper presents a procedure for the determination of displacements at any given point in a vibrating body which is based on the use of strain gauges. Knowledge of the modal properties of the body is necessary for the implementation of the method. Experimental or numerical evaluation of strain to displacement sensitivity is also needed. Simplicity of the required experimental apparatus, computational efficiency, and low cost are advantages of the proposed method.

To verify the method a clamped-end beam was instrumented and experimented upon under different loading conditions. The displacements derived with the proposed method show good agreement with displacement measurements performed with a Laser Doppler Vibrometer.

2. THEORETICAL DEVELOPMENT

Consider a general elastic, isotropic body. The problem we want to address is the evaluation of the displacement at any position $\bar{\mathbf{r}}$ in the body by means of few strain gauge transducers. We assume a general equation of motion

$$m(\mathbf{r})\frac{\partial^2\mathbf{u}(\mathbf{r},t)}{\partial t^2}+c(\mathbf{r})\frac{\partial\mathbf{u}(\mathbf{r},t)}{\partial t}+L(\mathbf{u}(\mathbf{r},t))=\mathbf{p}(\mathbf{r},t), \qquad (1)$$

where $\mathbf{r}$ is the position vector, m, c and $\mathbf{p}$ are the mass, the damping coefficient and the load per unit domain (volume/surface/length), respectively, $\mathbf{u}$ is the displacement vector in the body at time t and location $\mathbf{r}$ and L is a linear differential operator in the spatial variables. For a uniform beam, for example,

$$L=EI\frac{\partial^4}{\partial x^4}, \qquad (2)$$

where E is the modulus of elasticity and I is the moment of inertia of the cross section. For a thin plate

$$L=D\nabla^4=D\left(\frac{\partial^4}{\partial x_1^4}+2\frac{\partial^4}{\partial x_1^2\partial x_2^2}+\frac{\partial^4}{\partial x_2^4}\right), \qquad (3)$$

where D is the plate stiffness defined by

$$D=\frac{Eh^3}{12(1-\nu^2)}, \qquad (4)$$

h is the plate thickness and ν is Poisson's ratio.

We assume that we can find normal modes such that the j-th normal mode of the system for undamped free vibration is $\Psi_j(\mathbf{r})$ and the modes are normalized so that

$$\int_V m(\mathbf{r})\Psi_j^T(\mathbf{r})\Psi_k(\mathbf{r})\mathrm{d}V=m_j\delta_{jk}. \qquad (5)$$

Also we assume that the damping coefficient satisfies

$$\int_V c(\mathbf{r})\Psi_j^T(\mathbf{r})\Psi_k(\mathbf{r})\mathrm{d}V=c_j\delta_{jk}. \qquad (6)$$

This condition restricts the generality of the systems considered, see for example [5]. Finally, we assume that the solution to equation (1) can be given as

$$\mathbf{u}(\mathbf{r},t)=\sum_{j=1}^{\infty}\Psi_j(\mathbf{r})y_j(t). \qquad (7)$$

Substituting (7) into (1), multiplying both sides of the resulting equation by $\Psi_j^T(\mathbf{r})$, and integrating over the domain of the body, we obtain

$$\ddot{y}_j+\gamma_j\dot{y}_j+\omega_j^2y_j=\frac{1}{m_j}\int_V\Psi_j^T(\mathbf{r})\,\mathbf{p}(\mathbf{r},t)\mathrm{d}V \qquad (8)$$

where ω_j is the j-th undamped natural frequency of the body and γ_j is the modal bandwidth defined as

$$\gamma_j=\frac{c_j}{m_j}. \qquad (9)$$

Equation (8) is one of j uncoupled scalar linear differential equations, one for each mode of the system, which describe the motion in the j-th mode. Solution of these equations leads to the functions $y_j(t)$ which substituted in (7) give the displacement field in the body.

Similar to eq. (7) (see [6]), the strain field in the body can be expressed as

$$\varepsilon(\mathbf{r},t)=\sum_{j=1}^{\infty}\Phi_j(\mathbf{r})y_j^{\varepsilon}(t), \qquad (10)$$

where in this case $\Phi_j(\mathbf{r})$ is the *tensor* representing the j-th normal mode of the system. The strain field in the body can also be obtained from the displacement field by direct differentiation, that is

$$\varepsilon(\mathbf{r},t)=\frac{1}{2}\left(\frac{\partial\mathbf{u}}{\partial\mathbf{x}}+\left[\frac{\partial\mathbf{u}}{\partial\mathbf{x}}\right]^T\right)=\sum_{j=1}^{\infty}\frac{1}{2}\left(\frac{\partial\Psi_j(\mathbf{r})}{\partial\mathbf{x}}+\left[\frac{\partial\Psi_j(\mathbf{r})}{\partial\mathbf{x}}\right]^T\right)y_j(t). \qquad (11)$$

Comparing (10) with (11) it is possible to set

$$y_j^{\varepsilon}(t)=y_j(t) \qquad j=1,\ldots\infty \qquad (12)$$

and

$$\Phi_j(\mathbf{r})=\frac{1}{2}\left(\frac{\partial\Psi_j(\mathbf{r})}{\partial\mathbf{x}}+\left[\frac{\partial\Psi_j(\mathbf{r})}{\partial\mathbf{x}}\right]^T\right) \qquad j=1,\ldots\infty. \qquad (13)$$

From eqns. (10) and (12) we see that the strain gauge signal can be utilized towards the calculation of the functions

$y_j(t)$. These in turn can be used in eq. (7) to calculate the displacement at any point in the body.

To this end, suppose that we want to determine the displacement at location $\bar{\mathbf{r}}$, that the strain component ε_{kl} at locations $\mathbf{r}_m$, $m = 1,\ldots n$, are measured, and that the response of the body can be approximated by a finite number n of modes. Then we can write eqn. (7) and eqn. (10) as:

$$\mathbf{u}(\bar{\mathbf{r}}, t) = \sum_{j=1}^{n} \Psi_j(\bar{\mathbf{r}}) y_j(t) \tag{14}$$

and

$$\varepsilon_{kl}(\mathbf{r}_m, t) = \sum_{j=1}^{n} \phi_{jkl}(\mathbf{r}_m) y_j(t), \tag{15}$$

where $\phi_{jkl}(\mathbf{r}_m)$ is the (k, l) component of the tensor $\Phi_j(\mathbf{r}_m)$. Equation (15) is a linear algebraic equation in the n unknowns $y_j(t)$. If the $\phi_{jkl}(\mathbf{r}_m)$ were known, using n strain gauges would allow to solve for these unknowns. The $\phi_{jkl}(\mathbf{r}_m)$ could be calculated directly from (13) by numerical differentiation, but this method is very sensitive to measurement errors if the mode shapes $\Psi_j(\mathbf{r})$ are determined experimentally and then curve-fitted (see [7]). Another option is to consider the transfer functions between the displacement $u = \mathbf{e}^T\mathbf{u}$ at location $\bar{\mathbf{r}}$, with $\mathbf{e}$ the direction of the displacement which is to be determined, and the n measured ε_{kl} at the $\mathbf{r}_m$ locations. Namely, taking the Fourier Transforms of (14) and (15), considering only n modes, we have:

$$U(\bar{\mathbf{r}}, \omega) = \sum_{j=1}^{n} \mathbf{e}^T \Psi_j(\bar{\mathbf{r}}) Y_j(\omega) \tag{16}$$

and

$$\mathrm{E}_{kl}(\mathbf{r}_m, \omega) = \sum_{j=1}^{n} \phi_{jkl}(\mathbf{r}_m) Y_j(\omega) \qquad m = 1,\ldots n, \tag{17}$$

where the $Y_j(\omega)$'s are the Fourier Transforms of the $y_j(t)$'s. Thus we have:

$$\frac{U}{E_{kl}}(\bar{\mathbf{r}}, \mathbf{r}_m, \omega) = \frac{\sum_{j=1}^{n} \mathbf{e}^T \Psi_j(\bar{\mathbf{r}}) Y_j(\omega)}{\sum_{j=1}^{n} \phi_{jkl}(\mathbf{r}_m) Y_j(\omega)} \qquad m = 1,\ldots n. \tag{18}$$

Assuming that the modes of the structure are uncoupled, that is at each one of the resonant frequencies ω_j only the contribution of mode j is significant, (18) becomes

$$\frac{U}{E_{kl}}(\bar{\mathbf{r}}, \mathbf{r}_m, \omega_j) = \frac{\mathbf{e}^T \Psi_j(\bar{\mathbf{r}})}{\phi_{jkl}(\mathbf{r}_m)} \qquad m = 1,\ldots n; \qquad j = 1,\ldots n. \tag{19}$$

Therefore, with the mode shape functions $\Psi_j(\mathbf{r})$ known and the n transfer functions $\frac{U}{E_{kl}}(\bar{\mathbf{r}}, \mathbf{r}_m, \omega_j)$ measured, we can use (19) to calculate the coefficients $\phi_{jkl}(\mathbf{r}_m)$.

It is worth noting that, as an alternative to the experimental procedure outlined above, the $\frac{U}{E_{kl}}(\bar{\mathbf{r}}, \mathbf{r}_m, \omega_j)$ ratios could be calculated using the Finite Element method. This could be important in applications where the experimental analysis is difficult to carry out, an example being a space structure that cannot support its own weight in a 1 g gravity field.

3. PROPOSED METHOD

In order to obtain the displacement u at a generic location $\bar{\mathbf{r}}$ of a structure, the following steps are proposed:

1. Assume the response of the structure under dynamic loading to be accurately described by n modes of vibration only.
2. Perform a Modal Analysis to determine the n displacement mode shapes $\Psi_j(\mathbf{r})$.
3. Apply at least n strain gauges to the structure, each at a different location $\mathbf{r}_m$, to measure the strain component ε_{kl} at that location.
4. Determine the n transfer function values $\frac{U}{E_{kl}}(\bar{\mathbf{r}}, \mathbf{r}_m, \omega_j)$. This requires the displacement $u(\bar{\mathbf{r}}, t)$ to be measured.
5. Use the transfer function values to calculate the n^2 coefficients $\phi_{jkl}(\mathbf{r}_m)$ according to (19).
6. Finally use the strain signals under operating conditions to solve (15) for the $y_j(t)$'s. Substitute these values in (14) to get the desired displacement $u(\bar{\mathbf{r}}, t)$.

4. EXPERIMENTAL VERIFICATION

To verify the effectiveness of the proposed method two strain gauge bridges were attached to a 350 mm long clamped aluminum beam of rectangular cross section (50 mm x 3 mm), at distances 51 mm and 140 mm from the clamped end, respectively. A Polytec Laser Doppler Vibrometer was used to measure the displacement 335 mm from the clamped end. The strain gauge signals were used to reconstruct the measured displacements according to the derived methodology, when the beam was vibrating freely or when it was excited by the stinger of a Gearing & Watson MKII shaker. A sketch of the experimental set-up is shown in fig. 1.

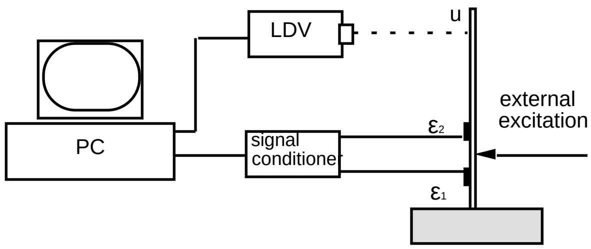

Figure 1. Experimental set-up.

Only the first two modes of vibration of the beam were taken into consideration. With reference to fig. 2, the first two natural frequencies of the beam are:

$$\omega_1 = 18.5\,\text{Hz} \text{ and } \omega_2 = 113\,\text{Hz}.$$

The mode shapes for a clamped beam are given by [8]:

$$\psi_j(x) = A_j\Big\{\big(\sin\beta_j L - \sinh\beta_j L\big)\big(\sin\beta_j x - \sinh\beta_j x\big) + \big(\cos\beta_j L + \cosh\beta_j L\big)\big(\cos\beta_j x - \cosh\beta_j x\big)\Big\} \quad j = 1, 2, \ldots, \infty, \tag{20}$$

where x is the coordinate along the axis of the beam, L is the length of the beam and

$$\beta_j^4 = m\frac{\omega_j^2}{EI}. \tag{21}$$

Normalizing the mode shapes using the maximum unity criterion, see [9], eqn. (20) gives

$$\psi_1(x = 335\,mm) = 0.937$$

and

$$\psi_2(x = 335\,mm) = -0.782\ .$$

Thus, from eqn. (14)

$$\mathbf{u}(x = 335\,mm, t) = 0.937\, y_1(t) - 0.782\, y_2(t)\,. \tag{22}$$

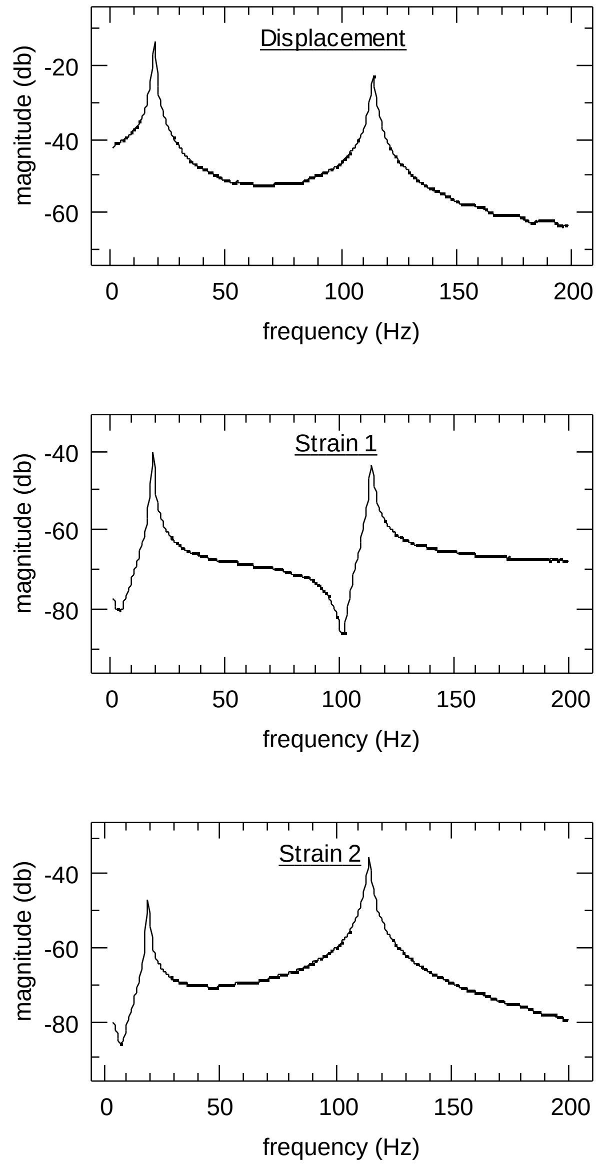

Figure 2. Power spectra for the displacement signal and the strain gauge signals at location 1 and location 2.

Eqn. (15) for the given set-up becomes

$$\begin{aligned}\varepsilon(\mathbf{r}_1, t) &= \phi_1(\mathbf{r}_1) y_1(t) + \phi_2(\mathbf{r}_1) y_2(t)\\ \varepsilon(\mathbf{r}_2, t) &= \phi_1(\mathbf{r}_2) y_1(t) + \phi_2(\mathbf{r}_2) y_2(t)\end{aligned}. \tag{23}$$

In order to use eqns. (23) to solve for $y_1(t)$ and $y_2(t)$ the coefficients $\phi_1(\mathbf{r}_1), \phi_2(\mathbf{r}_1), \phi_1(\mathbf{r}_2), \phi_2(\mathbf{r}_2)$ need to be known. For this simple case these coefficients could be determined by direct differentiation of equation (20). In a more general situation, however, the displacement mode shapes functions could be known only as the result of an experimental and curve-fitting procedure and, as previously stated, the differentiation process could lead to significant errors.

Furthermore, the scaling of the displacement functions would remain undetermined. The procedure outlined in the previous sections encompasses these problems. The transfer functions *U/E(ω)* relating the displacement at point $\bar{\mathbf{r}}$, measured with the laser doppler vibrometer, to the strains at location $\mathbf{r}_1$ and $\mathbf{r}_2$, measured with the strain gauges, are shown in figs. 3 and 4, respectively. Their values at the resonant frequencies are

$$\begin{aligned} \frac{U}{E}(\bar{\mathbf{r}},\mathbf{r}_1,\omega_1) &= 22.1, \\ \frac{U}{E}(\bar{\mathbf{r}},\mathbf{r}_1,\omega_2) &= -10.9, \\ \frac{U}{E}(\bar{\mathbf{r}},\mathbf{r}_2,\omega_1) &= 41.6, \\ \frac{U}{E}(\bar{\mathbf{r}},\mathbf{r}_2,\omega_2) &= 3.86. \end{aligned} \tag{24}$$

Eqns. (19) was then used to calculate the coefficients $\phi_1(\mathbf{r}_1), \phi_2(\mathbf{r}_1), \phi_1(\mathbf{r}_2), \phi_2(\mathbf{r}_2)$:

$$\begin{aligned} \phi_1(\mathbf{r}_1) &= 4.24\mathrm{E}-2, \\ \phi_2(\mathbf{r}_1) &= 7.17\mathrm{E}-2, \\ \phi_1(\mathbf{r}_2) &= 2.30\mathrm{E}-2, \\ \phi_2(\mathbf{r}_2) &= -9.46\mathrm{E}-2. \end{aligned} \tag{25}$$

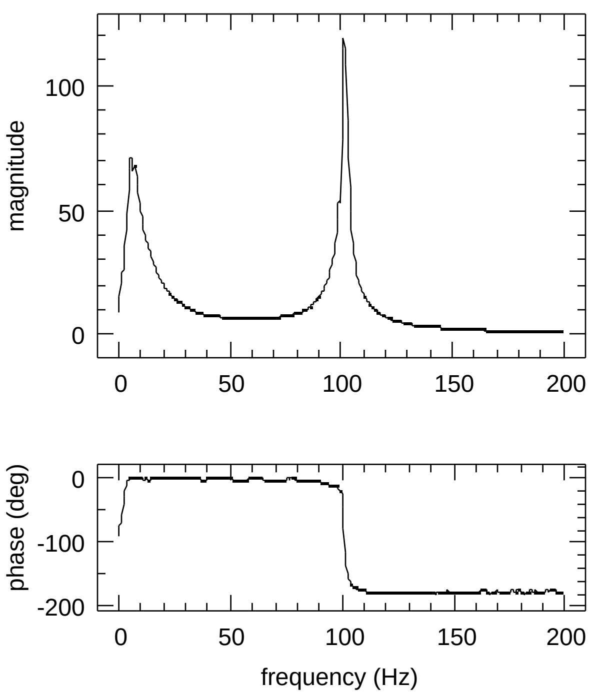

Figure 3. Transfer function between the displacement signal and the strain gauge signal at location 1.

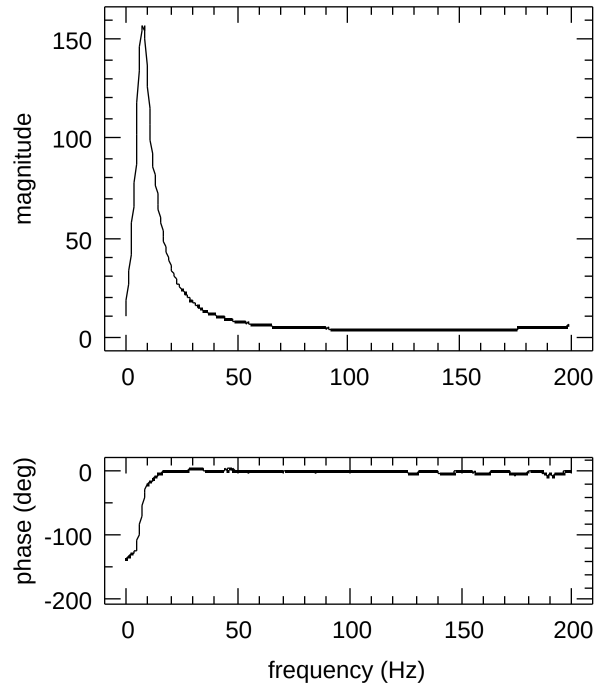

Figure 4. Transfer function between the displacement signal and the strain gauge signal at location 2.

These coefficients were in turn used during motion of the beam to solve eqns. (23) for $y_1(t)$ and $y_2(t)$. Finally, $y_1(t)$ and $y_2(t)$ were substituted in eqn. (22) to reconstruct the displacement signal.

Several loading conditions were considered. First, the vibratory response of the beam to a hammer impact applied 55 mm from its fixed end was measured. Figure 4 shows the measured and reconstructed displacement signals using unfiltered strain gauge signals. Figure 5 shows the result of an analogous experiment in which the strain gauge signals were filtered to eliminate any frequency component above 200 Hz. Then, in a forced excitation experiment, the beam was subjected to the double sinusoidal loading

$$F(t) = B_1 \sin(2\pi f_1 t) + B_2 \sin(2\pi f_2 t + \varphi). \tag{26}$$

The frequency values of the two sinusoids were first set at $f_1 = 25\,Hz$ and $f_2 = 105\,Hz$, see fig. 6, and then at $f_1 = 60\,Hz$ and $f_2 = 90\,Hz$, see fig. 7. Finally, the beam was excited driving the shaker with a broad band signal, see fig. 8.

5. RESULTS

For the free vibration case, see fig. 4, the reconstructed signal satisfactorily reproduces the measured displacement signal after the high frequency components originally present in the strain signals have died away. In fact, due to the higher dynamic sensitivity of strain in comparison to displacement, see [10], the strain signals contain frequency components above the second natural frequency of the beam which do not appear in the displacement signal. Inclusion of a third mode in the analytical development and use of a third strain gauge bridge station would allow to improve the quality of the reconstructed signal in the initial stages of the measurement also. Furthermore, as shown in fig. 5, when the strain gauge signals are filtered to eliminate all frequency components above 200 Hz, the reconstructed signal reproduces the measured displacement signal very well, even in the initial stages of the measurement. Only in the very beginning, when the hammer impacts the beam, a discrepancy can be noted.

With reference to fig. 6, which shows the measured and reconstructed displacement signals for the case of forced excitation with excitation frequencies close to the first two resonant frequencies of the beam, we notice that the two signals are almost identical. When the excitation frequencies are far from the resonant frequencies, namely for the second experiment using the forcing function described by eqn. (26), a difference in amplitudes between the reconstructed and the measured displacement signal can be noted, see fig. 7. This difference is due to the fact that the response of a structure in a frequency band which is not close to a resonant frequency is made up of contributions from several modes [11]. In the present work, however, only the first two modes were considered.

For the last case, see fig. 8, corresponding to random excitation, the measured displacement signal is buried in the reconstructed signal. Here again, the strain signals are much richer in frequency than the displacement signal. Disregarding these high frequency contributions, the measured and reconstructed displacements agree very well. As above, the addition of more strain gauge stations would improve the reconstruction.

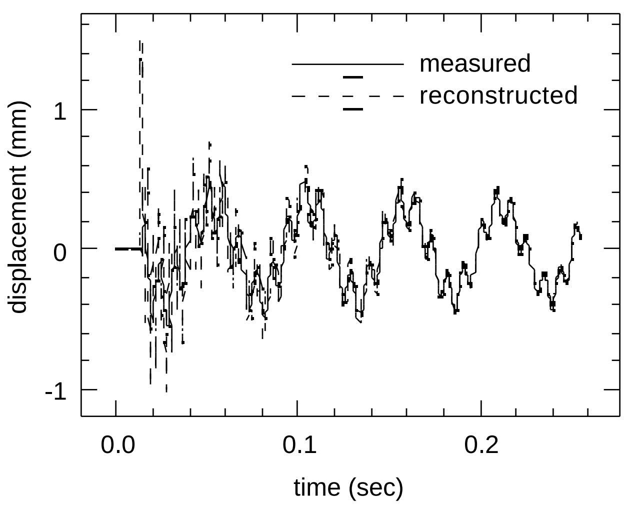

Figure 4. Measured and reconstructed displacements under free vibration of the beam, no filtering.

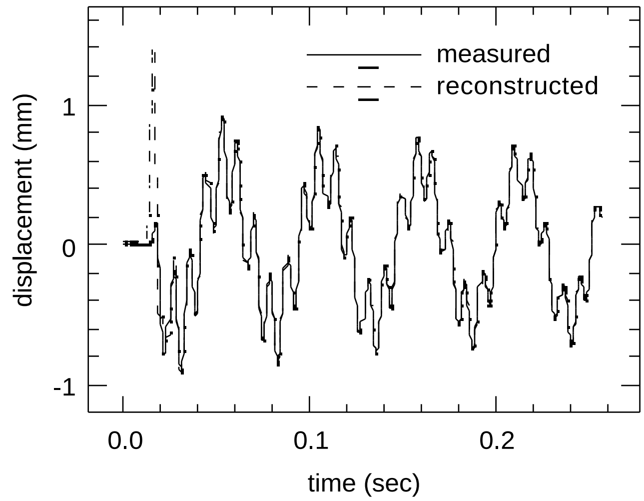

Figure 5. Measured and reconstructed displacements under free vibration of the beam, filtering above 200 Hz.

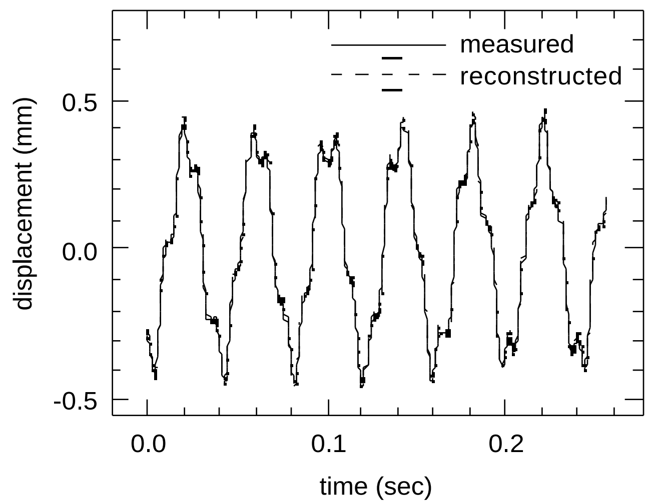

Figure 6. Measured and reconstructed displacements, double sinusoidal loading, f_1 = 25 Hz and f_2 = 105 Hz .

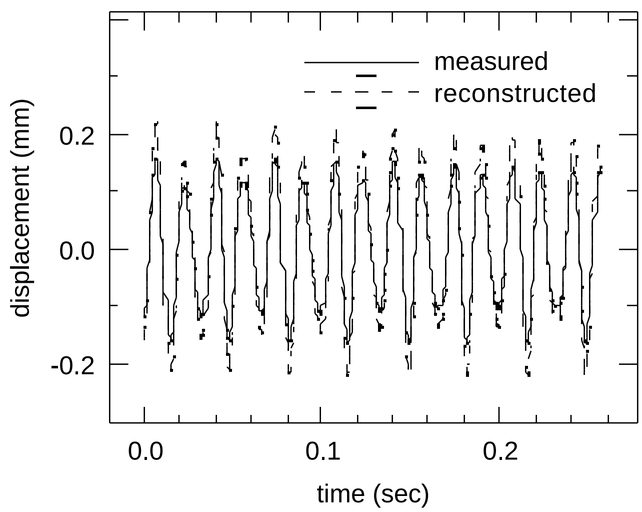

Figure 7. Measured and reconstructed displacements, double sinusoidal loading, f_1 = 60 Hz and f_2 = 90 Hz .

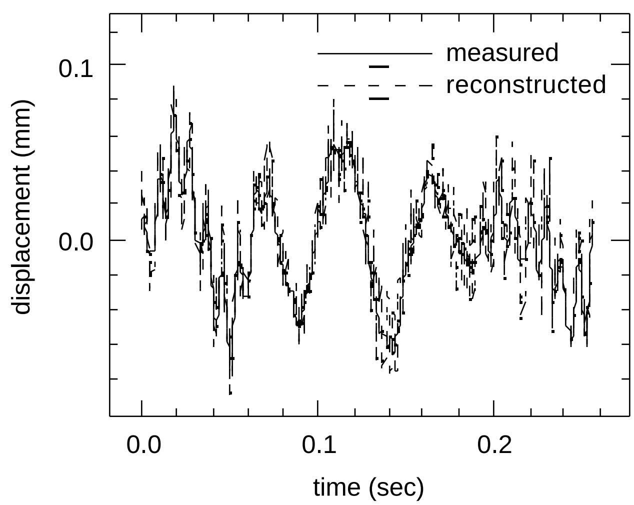

Figure 8. Measured and reconstructed displacements, random excitation.

8. CONCLUSION

In order to respond to the need for on-line displacement estimation in vibrating solids, to be applied for example in control schemes for the position control of flexible manipulators or for active vibration suppression in structures, a novel procedure based on the use of strain gauge measurements was developed. The procedure requires knowledge of the modal model of the structure. Assuming that the vibratory motion of the structure is accurately described by *n* modes only, at least *n* strain gauge stations are needed to reconstruct the displacement signal. Furthermore, the ratio of the displacement signal to the strain gauge signal at each resonant frequency of interest needs to be determined, either experimentally or by finite element techniques.

Some preliminary tests were performed on a clamped beam to verify the effectiveness of the method under different loading conditions. The results show good agreement between directly measured and reconstructed displacement signals. The discrepancies recorded for certain loading conditions are well explained considering the limitations introduced by using only two strain gauge stations.

The advantages of the method are its low cost and its computational simplicity which make it applicable to real-time control strategies. Additional experimental testing is needed to confirm the validity of the method when applied to more complex structures. An important issue is the choice of the strain gauge locations for optimal reconstruction and signal to noise ratio reduction.

9. ACKNOWLEDGEMENTS

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10. REFERENCES

[1] Dubowsky, S., "Dealing with Vibrations in the Deployment Structures of Space Robotic Systems", to be presented at the *5th International Conference on Adaptive Structures,* Sendai, Japan, December 1994.

[2] Cannon, R. H., Jr., Schmitz, E., "Initial Experiments on the End-Point Control of a Flexible One-Link Robot", *The International Journal of Robotics Research*, Vol. 3, No. 3, Fall 1984.

[3] Hollars, M. G., Cannon, R. H. Jr., Alexander, H. L., Morse, D. F., "Experiments in Advanced Control Concepts for Space Robotics: An Overview of the Stanford Aerospace Robotics Laboratory", *10th Annual AAS Guidance and Control Conference*, Keystone, Colorado, January 31-February 4 1987.

[4] Chalhoub, N. G., Zhang Xiaoying, "Reduction of the End Effector Sensitivity to the Structural Deflections of a Single Flexible Link: Theoretical and Experimental Results", *ASME Journal of Dynamic Systems, Measurement and Control*, Vol. 115, pp. 658-666, December 1993.

[5] Caughey, T. K., O'Kelly, M. E. J., "Classical normal modes in damped linear dynamic systems", *ASME Journal of Applied Mechanics*, Vol. 32, pp. 583-588, 1965.

[6] Bernasconi, O., Ewins, D. J., "Modal Strain/Stress Fields", *Journal of Modal Analysis*, pp. 68-76, April 1989.

[7] Liefooghe, D., Van Brussel, W., Sas, P., "Application of Dynamic Analysis Methods in Fatigue Lifetime Calculations", *Proceedings of the 15th International Seminar on Modal Analysis*, 19-21 September 1990, pp. 759-778.

[8] Humar, J. L., "Dynamics of Structures", Prentice Hall Inc., Englewood Cliffs, New Jersey 07632, 1990, pp. 668-672.

[9] CADA-X Modal Analysis, User Manual, LMS International.

[10] Hillary, B. Ewins, D. J., "The Use of Strain Gauges in Force Determination and Frequency Response Function Measurements", *2nd International Modal Analysis Conference*, Vol. 2, pp. 627-634, 1984.

[11] Ewins, D. J., "Modal Testing: Theory and Practice", Research Studies Press Ltd, John Wiley & Sons Inc., pp. 171-173, 1984.