ON THE LIMITATIONS OF PREDICTIONS OF THE DYNAMIC RESPONSE OF MACHINES WITH CLEARANCE CONNECTIONS

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ABSTRACT

The results of experimental and analytical studies of the dynamic response of machines with flexible links and connection clearances are presented. These results suggest that both a large amount of variability and high sensitivity to small parameter changes and operating conditions are inherent properties of the dynamic response of such systems. This work indicates that the accuracy of dynamic force predictions given by computer simulations for such systems may be fundamentally limited, and therefore such simulations should be used with care in system design.

1. INTRODUCTION

The high productivity, high technology systems demanded by todayÕs industry often require very high speeds and precision, but attempts to design machines with increased performance may result in unexpectedly poor dynamic behavior (Kakizaki et al., 1984). Typical problems encountered have included increased vibration and noise, reduced reliability and life, and, particularly, loss of precision. These problems often are due to the inability of computer-based design tools to include realistic properties in models of the dynamic behavior of complex machine systems. Useful design models should include important machine dynamic properties, such as the distributed mass and flexibility of machine elements and structures, and

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clearances in their connections. Practical modeling techniques also need to be computationally efficient and should be experimentally verified.

Figure 1 shows a typical realistic machine system. It has flexible components, an extended support structure and enclosure, and connections with clearances. Dynamic models for the design of such systems should be able to predict such characteristics as vibration, noise levels and internal bearing forces. This paper presents work on the development and experimental verification of analytical models for the design of these systems. As discussed below, the results suggest that the basic physics of these systems may limit our ability to predict accurately their dynamic behavior.

Engineers have long known that non-ideal machine properties such as clearance, backlash, the impacts they cause, and link flexibility often degrade machine dynamic performance (Johnson, 1963). Yet, to design machines without significant degrading non-ideal characteristics is frequently impossible, and nearly always expensive. The development of analytical models that could represent such non-ideal characteristics of machine systems, so that their effects could be controlled during the design process, has long been a goal of the mechanisms research community (Lowen and Chassapis, 1986; Thompson and Sung, 1986 ; Dubowsky and Maatuk, 1975).

Some of the earliest modeling methods that dealt with link flexibility used analytically tractable models, for example, Euler-Bernoulli beams, to represent the linksÕ flexibility (Lowen and Jandrasits, 1972; Dubowsky and Maatuk, 1975). Such approaches are limited to simply shaped links. Finite Element (FE) techniques were applied to model more generally-shaped links, first for planar (Bahgat and Willmert, 1976) and, later, for spatial systems (Sunada and Dubowsky, 1981). FE methods for machine systems have reached the level of commercial implementation (Wan and Haug, 1986).

Approaches to modeling the dynamic effects of non-ideal connections can be divided into three classes: those that model the joint as having compliance and friction but no clearance gap, and therefore no impacts; those that model the gap, but treat the resulting impacts as instantaneous events, characterized by conservation of momentum and a restitution coefficient, and are therefore unable to predict the impact force magnitudes; and those that model the internal contact forces of joints with clearance, compliance and friction. The first two approaches have been combined with link models using both rigid body and flexible body dynamics (Bagci, 1975; Haug et al., 1986; Khulief, 1986; Khulief and Shabana, 1986). These modeling approaches are relatively fast computationally. Since they cannot predict internal joint impact forces, they may not be appropriate for systems where the important effects include impact-excited vibration and noise in the machine system and its surrounding support and enclosure structures, and bearing failures due to excessive impact forces.

The third approach can predict a detailed time history of a joint’s internal contact forces, including impacts. The contact forces are nonlinear functions of a joint’s relative motions, and also depend on the joint’s internal geometry and construction details such as material properties. The method can be computationally expensive. It was first applied to simple linear machine systems (Dubowsky and Freudenstein, 1971). It was then applied to planar systems with rigid bodies (Dubowsky, 1974) and studied experimentally (Dubowsky and Young, 1975; Haines, 1985; Dubowsky et al., 1984). As the computational power of computers increased, the method was extended to modeling planar systems with flexible links (Dubowsky and Gardner, 1975; Dubowsky and Moening, 1978), and, combined with FE methods, it has also been applied to spatial, flexible link dynamic models

(Dubowsky et al., 1987; Kakizaki et al., to appear).

Systems with clearance impacts have been studied experimentally (Pfeiffer, 1992). Indirect measurements of bearing impact loads were reported (Cummings and Means, 1986). Some researchers have measured the accelerations of flexible links in systems that contain clearance connections. Aperiodic link accelerations in response to periodic inputs were observed (Stammers and Ghazavi, 1991). Large amplification factors, typical of systems with impacts, were observed (Soong and Thompson 1987). Experimental responses that differed qualitatively from numerical predictions have been noted (Liao et al. 1986).

A major assumption underlying the development of more detailed modeling methods is that finite computer speed and capacity are the limiting factors in attempts to improve the accuracy and valid frequency bandwidth of dynamic models. Recently, it has been suggested in the structural dynamics field that a more constraining limit may affect models of linear structures when they are excited by frequencies higher than the natural frequencies of their first few structural modes (von Flotow, 1987). In this paper, two systems, a simple impact beam and a spatial slider crank, are studied in depth both analytically and experimentally. The results suggest that, in systems with impact *nonlinearities*, such a non-computational limit may be active even for driving speeds well below the frequency of the first structural mode of the system. This limitation appears to be fundamental, related to the physics of such systems, not due to limitations of computer power or to modeling idealizations.

In the remainder of the paper, an analytical technique that has been developed to model spatial machine systems with flexible links and clearances is described. The experimental systems are presented, and results of the analytical and experimental studies are compared.

2. A BRIEF DESCRIPTION OF THE ANALYTICAL MODELING APPROACH

The modeling techniques used in this research were first developed for flexible, spatial machine systems with ideal joints (Sunada and Dubowsky, 1981), and later extended to include joints with compliance and clearance (Dubowsky et al., 1987; Kakizaki et al., to appear). The method uses Hartenberg-Denavit 4x4 transformation matrices to represent the nominal motions of each link in the system. The distributed mass and flexibility of the systemÕs links and support structure are described using Finite Element (FE) methods. Perturbation coordinates describe the motions of the FE node points with respect to the nominal motion frame of each flexible link. The dynamic equations of motion for each link are derived using LagrangeÕs formulation, using the perturbation coordinates as generalized coordinates. Component Mode Synthesis (CMS) is used to improve numerical efficiency. Kinematic compatibility matrices and force relationships express the constraints between the links and are used to assemble the dynamic equations of the system. The resulting global dynamic equations have the form:

$$[\mathbf{M}]\{\ddot{\mathbf{q}}\} + [\mathbf{G}]\{\dot{\mathbf{q}}\} + [\mathbf{K}]\{\mathbf{q}\} = \{\mathbf{Q}\} \quad (1)$$

The matrices $[\mathbf{M}]$, $[\mathbf{G}]$ and $[\mathbf{K}]$ describe the mass, damping and stiffness characteristics of the system and, in general, are time varying and nonlinear. The vector $\{\mathbf{q}\}$, and its derivatives $\{\dot{\mathbf{q}}\}$ and $\{\ddot{\mathbf{q}}\}$, are the global independent coordinates, velocities and accelerations. The vector $\{\mathbf{Q}\}$ describes the forces applied to the system, including actuator forces/torques, external loads, and internal bearing forces. These relationships are also nonlinear. The use of perturbation coordinates permits the kinematic nonlinearity terms to be transformed into time-varying terms for some systems (Sunada and Dubowsky, 1981). However, the nonlinear bearing forces due to joint clearances cannot be treated in this manner (Dubowsky 1974, Dubowsky et al. 1987).

The matrices [$\mathbf{M}$], [$\mathbf{G}$] and [$\mathbf{K}$], and the vector {$\mathbf{Q}$}, are functions of the link's FE mass and stiffness matrices, [$\mathbf{M}_i^{FE}$] and [$\mathbf{K}_i^{FE}$], damping matrices that also include gyroscopic terms due to the nominal motion, [$\mathbf{G}_i$], "gyroscopic stiffening" terms due to the nominal motion, [$\tilde{\mathbf{K}}_i$], the link CMS transformation matrices, [$\mathbf{A}_i$], the link compatibility matrices [$\mathbf{B}_i$] and their derivatives, and the machine's nominal motion joint variables, $\theta_i(t)$, and their derivatives. For example:

$$
\begin{aligned}
[\mathbf{K}] = & \sum_{i=1}^{NL} [\mathbf{B}_i]^T [\mathbf{A}_i]^T \left[\mathbf{K}_i^{FE} + \tilde{\mathbf{K}}_i \right] [\mathbf{A}_i] [\mathbf{B}_i] \\
& + \sum_{i=1}^{NL} \sum_{j=1}^{NL} [\mathbf{B}_i]^T [\mathbf{A}_i]^T [\mathbf{G}_i] [\mathbf{A}_i] \frac{\partial \mathbf{B}_i}{\partial \theta_i} \dot{\theta}_i \\
& + \sum_{i=1}^{NL} \sum_{j=1}^{NL} \sum_{k=1}^{NL} [\mathbf{B}_i]^T [\mathbf{A}_i]^T \left[\mathbf{M}_i^{FE} \right] [\mathbf{A}_i] \left(\frac{\partial^2 \mathbf{B}_i}{\partial \theta_j \partial \theta_k} \dot{\theta}_j \dot{\theta}_k + \frac{\partial \mathbf{B}_i}{\partial \theta_j} \ddot{\theta}_j \right)
\end{aligned}
\qquad (2)
$$

where NL is the number of links in the system. A computational block diagram of the technique, including clearance joint force computations, is shown in Figure 2.

Several clearance joint models have been produced in this work. For example, the spherical clearance connection model (SCC), shown in Figure 3a, models non-ideal spherical joints. The spatial revolute clearance connection (SRCC), not shown, can model non-ideal revolute joints. The spatial prismatic clearance connection (SPCC), shown in Figure 3b, is a model for non-ideal prismatic sliding joints. Each joint model consists of nonlinear functions that define components of the system force vector, {$\mathbf{Q}$} in Equation 1, in terms of the internal motions of the joint, the geometry of the joint type, and parameters such as the joint clearance, materials and dimensions.

The SRCC and the SPCC may have simultaneous contact at several points, determined by the details of the complex, spatial motions of their joint halves. The SCC and SPCC analytical models are discussed in Section 4.

The analytical method and joint models have been implemented in a software package called ASSET (*A*dvanced *S*patial *S*ystems *E*mulation *T*echnique). This software has been used to study a number of machine systems, including a robotic manipulator (Kakizaki et al., to appear), and the two systems discussed in this paper, the impact beam system and the spatial slider crank.

3. THE IMPACT BEAM SYSTEM

A. System Description

The simple impact beam system (IBS) consists of a beam held at one end by a zero-clearance bearing, see Figure 4. At the other end of the beam is a one-dimensional clearance joint with adjustable clearance. The beam is driven by an external force, F(t) in the figure. The beam end at the clearance connection moves back and forth in the clearance joint, generating impact forces. The assembly is mounted to a base plate, and, optionally, enclosed by a cover. Each of the elements of the IBS is designed to represent a feature typical of machine systems: the beam represents a flexible link; the clearance connection represents a bearing with internal clearance; the base plate and cover represent the support structure and enclosures. The purpose of the IBS is to permit the study of clearance impacts without interactions among multiple nonlinearities; therefore, its design excludes nonlinear nominal kinematic motions and multiple clearance connections. Despite the simplicity of the IBS, the results from its experimental and analytical models are shown below to provide some interesting and possibly important insights for the behavior of machine systems with internal impacts.

B. An Experimental IBS

An experimental version of the IBS, shown in Figure 5, was constructed. The experimental IBS uses either an aluminum or a steel beam. Two covers were fabricated for the IBS. One is made of aluminum panels screwed to a steel frame, and the other is a one-piece fiberglass design. The dimensions of these parts are listed in Table 1, with reference to the axes shown in Figure 5. An electrodynamic shaker provides the driving force.

The mounting methods of the steel and aluminum beams differ. The aluminum beam is cantilevered from the beam support. The steel beam is mounted to the beam support using a steel flexure pivot. Either method provides a zero-clearance bearing without impacts.

Table 1. IBS Construction.

Element	x-dimension	y-dimension	z-dimension
Al. Beam	28 cm (11 in)	1.9 cm (0.75 in)	0.318 cm (0.125 in)
Steel Beam	28 cm (11 in)	1.9 cm (0.75 in)	0.635 cm (0.25 in)
Al. Base plate	39 cm (15.4 in)	30.5 cm (12 in)	1.27 cm (0.5 in)
Cover	40.1 cm (15.8 in)	32 cm (12.6 in)	25.9 cm (10.2)
Flexure Pivot	0.4 cm (0.16 in)	1.9 cm (0.75 in)	0.0127 cm (0.005 in)

A close-up of the IBSÕs impact yoke structure and instrumented clearance joint is shown in Figure 6. Clearances used in these studies ranged from 0 (no clearance) to ±0.5 mm (±0.020 in). This range is typical of the clearances found in many machine systems. Clearances are always listed as ±1/2 the total free motion of the end of the beam. The DC component of the shakerÕs drive current was adjusted to assure that the beam end was precisely centered between the force sensors when at rest, compensating for gravity forces and alignment errors.

The impedance head shown in Figure 6 provides an indirect measurement of the contact forces. This signal is accurate for low frequencies, but was found to be dominated by the dynamics of the yoke at frequencies above 400 Hz. Therefore, the piezoelectric contact force sensors shown in the figure were developed to measure directly the impact forces at their tips, and were found to be accurate for frequencies up to at least 10,000 Hz. All impact force data presented in this paper were collected using these force sensors. Accelerometers were mounted near the center of the base plate and the top panel of the cover, and near one of the corners of the cover, to monitor the structural vibrations of the IBS. Data collection for the IBS was done with a General Radio 2215 16 channel FFT digital analyzer, useful for data with frequency content up to 25 kHz. A Nicolet 3021 digital storage oscilloscope was also used. Other hardware used with the IBS included PCB 462A charge amplifiers, a Bruel and Kjaer 2702 amplifier to drive the shaker, and Wavetek and Spectral Dynamics signal generators.

C. Analytical Models of the IBS

Using ASSET, IBS analytical models of varying complexity were constructed. The simplest is a rigid base model, in which the base plate, beam support post, and the yoke holding the force sensors, shown in Figure 5, are treated as rigid. The cantilevered mounting of the aluminum beam is modeled as a clamped boundary condition. The flexure pivot mounting of the steel beam is modeled as infinitely stiff for motions in all directions except the bending and translation driven by the shaker. The beam itself is represented by a finite element model using 37 beam elements approximately 7.6×10^{-3} m (0.3 in) long.

A more complete flexible base model was also implemented. It adds FEM models for the impact yoke, shaker connecting rod, beam support and base plate to the system. The base plate boundary conditions are taken as clamped at the four corners, and free elsewhere along the edges.

The beam’s FE model includes a node at the point where the force sensors touch the beam. The contact force is a nonlinear function of the motions of this node point. The contact force function includes a no-contact, zero force part for displacements smaller than the clearance gap, and a contact part with compliance and damping for displacements exceeding the gap. The stiffness coefficients are calculated using a linearized Hertzian contact analysis, and varied according to the beam construction, as listed in Table 2.

Table 2. Contact Parameters in the IBS Analytical Models.

	Aluminum Beam	Steel Beam
Contact Stiffness	$9.6 \times 10^{6} \frac{\text{N}}{\text{m}} \left(5.5 \times 10^{4} \frac{\text{lbf}}{\text{in}}\right)$	$1.5 \times 10^{7} \frac{\text{N}}{\text{m}} \left(8.4 \times 10^{4} \frac{\text{lbf}}{\text{in}}\right)$
Contact Damping	$7.0 \frac{\text{N-s}}{\text{m}} \left(0.040 \frac{\text{lbf-s}}{\text{in}}\right)$	$20 \frac{\text{N-s}}{\text{m}} \left(0.11 \frac{\text{lbf-s}}{\text{in}}\right)$

D. Impact Beam System Dynamic Response

The impact forces measured by the force sensors in the experimental IBS were compared with impact forces predicted by the analytical IBS models. For all results presented here, the driving force provided by the shaker was sinusoidal with ±9 N (2 lbf) amplitude.

Figure 7 shows calculated and experimentally measured bearing force histories for a half cycle of a 5 Hz shaker driving force. The steel beam was used, with ±0.127 mm (±0.005 in) bearing clearance. The rigid base analytical model was used to generate Figure 7b. These results show excellent agreement between the calculation and experiment, both qualitatively and quantitatively. The peak magnitude of the initial impact force and the overall shape of the curve, including resonant vibrations of the beam as it rests against the contact point, are well predicted by the simulation.

The time histories in figures 7a and 7b show the form typical of impact events. There is an initial, large impact force. This initial force is followed by a period of bouncing in and out of contact, ending at about 0.020 seconds. For the remainder of the half cycle, the beam is steadily in contact with its bearing. The contact force during this period shows a half cycle of the sinusoidal driving force with superimposed decaying oscillations that are due to structural resonances. At about 0.1 seconds, the beam loses contact, and a similar force profile occurs for the other side of the bearing after the beam crosses the clearance gap. Without impacts, the peak contact force, due to the 5 Hz driving force alone, would be about 6.2 N (1.4 lbf). The impact force, however, is about 34 N (7.6 lbf). The impact force is more than 5 times the nominal force. This amplification of nominal bearing forces due to impacts is well known and can have important degrading effects on system performance (Dubowsky and Freudenstein, 1971; Dubowsky et al., 1984; Dubowsky and Gardner, 1975; Kakizaki et al., to appear).

Figure 8 is produced by plotting initial peak forces, as identified by a circle in Figure 7, for various clearances while using the steel beam and a 5 Hz drive frequency. The rigid base model was used for the calculated results shown in this figure. The agreement between prediction and experiment is good overall, although it is somewhat better for smaller clearances. A trend of increasing peak impact force with increasing clearance is clearly predicted by the simulation and verified in the experiment. Note that at the largest clearance, ±0.508 mm (0.020 in), the predicted impact force is about 67 N (15 lbf), over 10 times the maximum nominal force of 6.2 N (1.4 lbf). The measured impact force is even higher. Comparable results for the aluminum beam with 5 Hz drive frequency, not shown, also demonstrate good ability of the simulation to predict the important impact forces. Impact forces at a given drive frequency and clearance setting are lower for the aluminum beam than for the steel beam, reflecting its lower mass and softer surface properties. This effect is both observed in the experiment and predicted by the simulation.

Figure 9 shows peak impact forces vs. clearance for the steel beam driven at 20 Hz. In this case the experiment and the numerical model do not agree as well as they did for a 5 Hz force input, although the fundamental trend is still predicted. Comparable results for the aluminum beam, not presented here, show similar predictive ability of the simulation with respect to the basic trend, and similar reduced accuracy of prediction for larger clearance values. While in Figures 8 and 9 the calculated force tends to be lower than the experimental result, this is not the rule for all the cases studied. Often, the experimentally measured forces are lower than the calculated ones.

The reluctance of the experiment to be modeled accurately at higher frequency led to a series of *experiments* using the IBS in which the clearance and beam construction were held constant, and the shaker drive frequency was varied in small, precisely controlled increments over a narrow frequency range. The shaker frequency was provided by a stable analog source, and its period was monitored to maintain a stability of 2 parts in 10,000 or better during data collection.

Figure 10 shows the interesting results obtained from the steel beam, with ±0.127 mm (±0.005 in) clearance and 27 drive frequencies bracketing 20 Hz. Data were collected in ensembles of 15 impact peaks at each frequency point. The solid curve in the figure plots the means of these ensembles of experimentally measured impact peaks, and the error bars show ±1 standard deviation for each ensemble. The dashed curve shows the results of the rigid base simulation model. This figure shows how sensitive the important impact forces can be to small changes in parameters. For a small change in shaker drive frequency, from 19.5 Hz to 21 Hz, the mean impact force changes from approximately 31 N (7 lbf) to 80 N (18 lbf), an increase of 150%. It should be noted that the lowest natural frequency of the steel beam, with free-free end conditions, is approximately 430 Hz, well above the approximately 20 Hz drive frequency used in this series of experiments.

The standard deviations of the peak impact force ensembles, shown by the error bars, change as markedly as their means. The standard deviation at 19.5 Hz is about 10% of the mean; at 21 Hz, it is approximately 33% of the mean. Thus, the peak experimental impact forces at 21 Hz are larger, on average, but less consistent from cycle to cycle than at 19.5 Hz. It should be emphasized that great care was taken in these experiments to assure that the operating parameters were held stable at each drive frequency. The IBS is able to produce highly varying impact forces in response to highly stable inputs.

Other experiments for different operating conditions and system parameters gave similar results. For the steel beam driven by closely-spaced frequencies bracketing 5 Hz, the mean impact forces show somewhat less variability as the drive frequency is changed, around 60% rather than 150%. This degree of variation still shows a surprising sensitivity of the impact force to small changes in drive frequency. Note that these are experimental observations, in no way numerical artifacts of the analytical model. Standard deviations of the impact force ensembles for drive frequencies bracketing 5 Hz are consistently smaller than those shown in Figure 10.

The conclusion that appears to emerge from these studies is that, while the impact beam system exhibits underlying trends similar to those seen in many earlier studies (Dubowsky and Freudenstein, 1971; Dubowsky and Gardner, 1975), namely that the impact forces increase as clearance and drive frequency are increased, it is clear that these trends are not smooth curves. The average trends exhibit substantial and important local variability. In addition, there is substantial variation in impact peaks from one cycle to the next of the input force, even when care is taken to assure stable operating conditions.

The sensitivity of the average impact forces to operating conditions may be due to the nonlinear dynamics introduced by the clearance. Linear systems always respond at the frequencies of their excitations. This is not true of nonlinear systems, especially when the nonlinearity is a clearance with impacts. Impact forces generated at a clearance connection have a nearly impulsive nature with a broadband, high frequency spectral characteristic. In the IBS, these impact forces excite a linear elastic beam and supporting structure, driving many high frequency modes of vibration in complex ways. Figure 11 shows a frequency-averaged autospectrum of the force sensor output for a series of impacts with 20 Hz shaker drive frequency and ±0.127 mm clearance, using the steel beam. The highly nonlinear nature of the clearance connection ÒsmearsÓ the pure-tone 20 Hz shaker force input very broadly in the frequency domain, so that there is significant excitation even at frequencies above 10 kHz, the 500^{th} harmonic of the input frequency. As the frequencies of peaks in the impact force spectrum change relative to the frequencies of structural modes in the IBS, the structure may respond to small changes in shaker drive frequencies with large changes in the structural vibration levels excited by the impact forces. This effect may be responsible for the high sensitivity of the mean impact force to drive frequency, observed in Figure 10.

The high frequency excitation provided by impact forces may contribute to the relatively poor performance of the numerical simulation seen under certain conditions. The difficulty of accurately modeling the response of even a single structural mode excited near resonance is well known, and impact force excitation is likely to excite several modes of structural vibration near their resonances simultaneously.

In addition, some amount of variability or uncertainty in the parameters of any physical structure is unavoidable, caused by manufacturing tolerances, for example.

Even for linear elastic structures, a mathematical model that appears to be accurate may give incorrect predictions for a real system because of this uncertainty. The ability to make such a model accurate for high frequencies degrades when the mode shapes and natural frequencies of the structure become sensitive to parameter variations on the scale of the uncertainty (von Flotow, 1987). As shown in Figure 11, impacts make the high frequency dynamics important in the total system response.

The combination of these effects means that even a Òvery accurateÓ analytical model may not duplicate the behavior of an experimental system, and the real behavior of a manufactured example of a machine system with clearances is unlikely to match closely the behavior predicted for it by analytical design tools.

The large observed cycle-to-cycle variations in impact forces, indicated by the large standard deviations, could be due to structural vibrations that do not decay between impacts. The magnitude of an impact force peak is strongly affected by the relative velocity of the impacting members just before contact. This velocity is partly caused by structural vibrations. Since these vibrations are not harmonically related to the basic driving frequency, they may cause non-periodic variations in the impact forces. The acceleration of the base plate is a sensitive indicator of system vibration. Figure 12 shows measurements of the base plate acceleration for 5 Hz and 20 Hz driving frequencies. Both Figure 12a and 12b are for the steel beam with ±0.127 mm clearance. Clearly, the systemÕs vibrations decay almost entirely between impacts with 5 Hz drive frequency, and they do not decay between impacts at 20 Hz. This characteristic mechanism may account for the higher standard deviations observed in impact force peaks at the higher drive frequency. The question of whether this non-periodic force variation is mathematically chaotic behavior cannot be ignored (Stewart, 1989), and is currently under investigation. In any event, this behavior has some important implications for machine designers, because it contributes to the difficulty of constructing accurate numerical models.

The IBS has two covers that can easily be changed or removed. The above results have been for the IBS with no cover. Figure 13 shows experimentally measured peak impact forces with the covers installed. There is no significant difference in impact forces between the aluminum and fiberglass covers. When operation of the IBS with either cover is compared with operation with no cover, a small effect on impact forces can be observed. The covers reduce the impact forces slightly at a driving frequency of 20 Hz, and increase them slightly at 5 Hz. While the covers have a fairly small effect on the impact forces, their effect on the sound radiated by the IBS is strong. A discussion of the noise generated by the IBS is beyond the scope of this paper, and may be found in (Oppenheimer, 1992).

The flexible base ASSET model can predict the effects of changing the base plate and cover. A cover can be added to this model as an extension of the FE model of the base plate. Table 3 compares typical results from the rigid base model and flexible base model without a cover, for the steel beam at 5 Hz driving speed and two clearance values.

Table 3. Peak Impact Forces.

Clearance	Rigid Base Model	Flexible Base Model
±0.254 mm (±0.010 in)	49.4 N (11.1 lbf)	37.8 N (8.5 lbf)
±0.508 mm (±0.020 in)	64.9 N (14.6 lbf)	53.8 N (12.1 lbf)

As expected from previous work, introducing additional compliance into the system model, in this case, that of the base plate, reduces the predicted impact forces (Dubowsky and Gardner, 1975). The basic trend of increasing impact force with increasing clearance is still seen.

The base plate vibrations predicted by the flexible base model, not shown, did not agree well with the measurements. Part of our current work focuses on developing a more computationally practical approach to modeling these vibrations well (Oppenheimer, 1992). The flexible base model requires an order of magnitude more computer time than the rigid base model.

To determine whether the behaviors seen in the IBS were also present in a more complex, realistic machine system, a spatial slider crank mechanism was studied experimentally and analytically, as discussed below.

4. THE SPATIAL SLIDER CRANK MECHANISM

A. System Description

The spatial slider crank (SSC) is typical of many real machine systems with multiple links, large, fully spatial nonlinear kinematic motion, and clearance connections between the links. The SSC used in this study is shown in Figure 14. A limited-motion ball joint connects the motor-driven crank to the connecting rod, and a universal joint connects the connecting rod to the slider. There is a prismatic joint between the slider and the guide rod. A spatial slider crank is a generalization of the well-known planar slider crank mechanism. In a spatial slider crank, the rotation axis of the crank need not lie in a plane perpendicular to the slider’s axis. When the angle between the crankÕs axis and the sliderÕs axis is different from 90 degrees, the spatial slider crank has fully spatial kinematics.

B. The Experimental Spatial Slider Crank

The experimental SSC, Figure 14b, uses a flywheel on its crank to help stabilize the speed of the mechanism. The ball joint and the sliding prismatic joint were implemented in this apparatus as instrumented clearance joints with adjustable clearance; these joints are illustrated in Figure 15. Some of the important dimensions of the SSC are listed in Table 4.

Table 4. SSC Dimensions.

Crank radius	5 cm (2 in)
Connecting rod	0.64 cm dia. x 19 cm long (0.25 x 7.5 in)
Slider guide rod	Square, 1.3 cm side (0.5 in)
Motor angle range	90 to 65 degrees
Slider stroke	10 to 9.2 cm (4 to 3.6 in)
Joint clearances	near 0 to more than 1 mm (0.040 in)
Aluminum base plate	56 x 46 x 1.3 cm (22 x 18 x 0.5 in)

The internal contact forces of the adjustable-clearance joints are measured using miniature versions of the force sensors developed for the IBS. The instrumented joints are lubricated with light machine oil. The spatial slider crank runs at speeds from 50 to 250 rpm. Data are collected from the spatial slider crank using a Concurrent 6000 computer with high speed data acquisition hardware and software. This equipment can simultaneously acquire 16 channels of data; simultaneous multichannel data acquisition is important for the SSC because its joints can have multiple points in contact at any instant. Signal conditioning for the force sensors is performed by Endevco, B&K and PCB charge amplifiers and a Frequency Devices 9016 multichannel programmable analog filter. Other equipment includes a variable regulated DC power supply for the spatial slider crankÕs motor, monitoring equipment for the motorÕs integrated tachometer, and a photonic sensor to acquire a once-per-revolution phase marker from the flywheel.

C. Analytical Model of the Spatial Slider Crank

The analytical model of the spatial slider crank uses the spherical clearance connection model, the SCC, to represent the ball joint, Figure 3a, and the spatial prismatic clearance connection model, the SPCC, for the slider joint, Figure 3b. The SCC is a spherically symmetric model with an inner spherical ball and an outer spherical socket, and it permits at most one point of contact at any time. Contact may occur anywhere on the inner surface of the socket. The SPCC is an axisymmetric model with eight possible contact points located at fixed coordinates on the slider shell, placed to simulate the force sensors of the instrumented sliding joint shown in Figure 15b. Zero to six points on the SPCC may be in contact at any time. Forces at each contact point in both joint models are represented by nonlinear functions that have a zero force non-contact region, and linearized Hertzian contact compliance with linear damping after contact occurs. The parameters used are listed in Table 5. Details of the contact force modeling procedure are beyond the scope of this paper, see (Dubowsky et al., 1987) and (Deck, 1992). The connecting rod is represented as a flexible link using FE methods, with CMS modal truncation performed at 5000 Hz, 8 modes, to reduce the model's order. The flywheel, yoke, slider guide rod and base plate are modeled as rigid, and the motor is assumed to rotate at constant speed.

Table 5. SSC Numerical Model Contact Parameters.

	SCC joint	SPCC joint
Contact Stiffness	$2.8 \times 10^7 \frac{N}{m} \left(1.6 \times 10^5 \frac{lbf}{in}\right)$	$1.4 \times 10^7 \frac{N}{m} \left(8.2 \times 10^4 \frac{lbf}{in}\right)$
Contact Damping	$44 \frac{N\text{-}s}{m} \left(0.25 \frac{lbf\text{-}s}{in}\right)$	$51 \frac{N\text{-}s}{m} \left(0.29 \frac{lbf\text{-}s}{in}\right)$

D. Dynamic Response Results

Figure 16 shows the contact force measured experimentally at one of the vertical force sensors on the slider during a 10 second period while the SSC was being operated at 230 rpm. About 38 cycles appear on the figure, and there is a distinctive once per cycle force peak caused by impacts. As with the IBS, there is a great deal of non-periodic variability in impact forces as the SSC goes through cycles of motion at constant speed. The peak contact force shown in Figure 16 varies between approximately 27 N (6 lbf) and 80 N (18 lbf). Expanding the time scale shows that impacts in the spatial slider crank follow a pattern similar to those seen in Figure 7a: an initial, large and very short peak, a short period of bouncing, and then a period in contact during which both the nominal force and superimposed structural resonance components may be noted in the contact force. The analytical results, not shown, reveal the basic trends of increasing impact force as motor drive speed or clearances increase (Deck, 1992). They also show some cycle to cycle variation in the peak impact force.

Figure 17 indicates the trends in peak impact forces and cyclic force variations typical of the SSC. The SSC was operated at different speeds, and contact forces at one of the prismatic joint’s force sensors were recorded. The upper line shows the largest peak force observed during several cycles of operation at each speed. This line thus shows the trend in peak impact forces. The lower line shows the smallest peak force observed from the other cycles in the data at approximately the same crank angle as the largest force. The separation of the two lines thus indicates the degree of cyclic force variability. A filled circle indicates that the force was caused by an impact. Figure 17 shows that when there are no impacts, the peak forces are consistent from cycle to cycle, and the maximum and minimum force lines are nearly the same. As the speed of the SSC is increased, impacts in the prismatic joint begin between 150 and 200 rpm. Once impacts begin to occur, the peak forces increase dramatically, and in addition there is variation in the peak force from one cycle to the next. Thus, the lines diverge. As in the IBS, impact forces in the SSC are not consistent during cycles of “steady-state” machine operation.

Analytical studies of the SSC illustrate the sensitivity of this system to small changes in its parameters. For example, changing the modeled position of the contact points on the slider by less than 5×10^{-10} m (2×10^{-8} in) from nominal dramatically changes the times of impact predicted by the simulation. Such effects would not occur in a well-behaved linear system. Small changes in the coefficient of sliding friction used in the prismatic joint contact force model also produce large changes in the predicted impact forces. In fact, under some conditions, neglecting prismatic joint friction in the numerical model results in a prediction of no impacts, contrary to the experimental results. Such results show the caution that is necessary when creating and interpreting analytical results for these systems. A designer who did not include sliding friction in the analytical model, not an unreasonable assumption for a well lubricated joint, might mistakenly conclude that the system would not exhibit joint impacts.

5. CONCLUSIONS

The results presented in this paper suggest that high sensitivity to parameters and large variations in impact forces may be natural, fundamental characteristics of the physics of machine systems with clearance impacts and significant flexibility in their components. This behavior has been found even in a very simple mechanical system, the impact beam system; it is clearly present in both the calculated and the experimentally measured response of this system, and hence is not a computational artifact. It has also been observed experimentally and analytically in a more realistically complex system, the spatial slider crank. This behavior might in part be caused by the machine’s elastic response to excitation by the broadband, high frequency, nonlinear impact forces within its joints. Whether this behavior is chaotic behavior, in a mathematical sense, is an interesting question currently being investigated.

In any event, this behavior has important implications for the modeling community as it attempts to develop computer-based tools for the design of such systems. Meaningful predictions of machine system performance, such as the maximum bearing forces, may require a large number of simulations to be performed with varying operating conditions and system parameters. Designers who use analytical tools to predict the dynamic performance of such systems should be aware that they are obtaining at best a rough estimate of the nature and magnitude of impact forces that the real system will experience.

6. ACKNOWLEDGMENTS

The contributions of Charles Oppenheimer, Erin OÕConnell and Uwe Müller in the design and construction of the experimental systems are acknowledged, as well as the contributions of Peng-Yun Gu to the computer studies. Finally, the support of the National Science Foundation under grant number MSS 9023487 is gratefully acknowledged.

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