

# Path-Planning for Elastically Constrained Space Manipulator Systems

Miguel A. Torres and Steven Dubowsky

Department of Mechanical Engineering  
Massachusetts Institute of Technology  
Cambridge MA 02129

## Abstract

Current design proposals of space manipulator systems suggest the use of a general-purpose dextrous arm attached to the end of a long flexible arm to perform a variety of tasks. The dynamic interaction between the components of such systems present a number of dynamic and control problems. The motion of the small manipulator can excite difficult to control vibrations of the large manipulator. This paper presents a technique called the Coupling Map to plan motions on elastically constrained space manipulator systems that result in relatively low residual vibrations to its supporting structure. A prototype was designed in some detail as a baseline for simulation and evaluation of the algorithms developed in this study. Experimental evaluation of these algorithms is currently underway.

## 1 Introduction

Telerobotic manipulator systems have been proposed as an alternative to costly and hazardous Extra Vehicular Activities. These systems are expected to perform, more efficiently and precisely, a variety of tasks which may range from servicing a satellite or a spacecraft in orbit to assembling space structures [3]. Systems like the Space Station Remote Manipulator System (SSRMS) and the Special Purpose Dextrous Manipulator (SPDM) have been proposed to meet these objectives [6,8], see Figure 1. In this configuration, the low-bandwidth, less accurate SSRMS will provide a large working envelope while the wider-bandwidth, more accurate SPDM will provide fast and precise motion and forces.

However, motion of the small high-bandwidth SPDM can excite low frequency vibrations in the large SSRMS, making it necessary to solve the problem of such vibration before this system can be effectively utilized. Operating the SPDM slowly does eliminate this problem, but at the cost of reducing overall system effectiveness by greatly increasing the time required to perform tasks. The current Remote Manipulator System, or RMS, of the Space Shuttle must be operated slowly in order to avoid dangerous vibrations; an astronaut must wait between 20 and 40 seconds for the manipulator arm to settle after a move [14]. This prevents the system from capturing even a slowly spinning satellite, as shown in a recent well published event

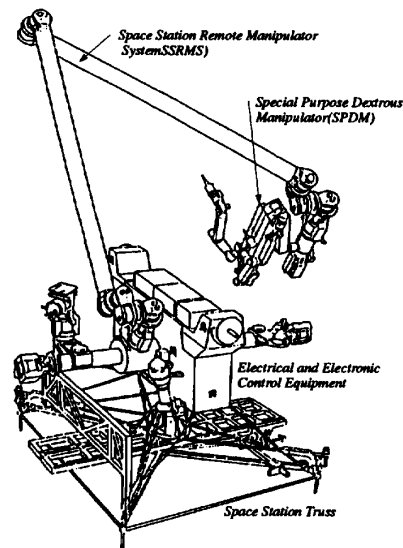


Figure 1: The Proposed Space Station Remote Manipulator System (SSRMS) and Special Purpose Dextrous Manipulator (SPDM). (Illustration provided by NASA Langley Automation Technology Branch)

[5]. Based on current design specifications, similar vibrational problems are expected from the proposed SSRMS, particularly when excited by the motion of the two-arm 19 degree-of-freedom SPDM.

In many tasks the large SSRMS will be stationary, its joints locked, while the small SPDM performs its functions. It is expected that this mode of operation will be an operational constrain placed on the system which will allow the system to be modeled as a rigid redundant manipulator mounted on a highly flexible supporting structure.

The literature that focuses on the modeling and control of *rigid* space robots, particularly free-flying or free-floating systems, is relatively large [11,18,19]. A number of studies of space robots focus on the modeling of *flexibility* in their links and to a lesser degree

the control of this flexibility [2,10,12]. One method which has received some attention is based on pre-filtering to reduce vibrations [14]. However, while this method works successfully with linear systems like simple one-link manipulators, relatively few studies have treated problems of *flexibly* supported manipulators. These studies have largely treated problems in terrestrial or industrial systems, focusing on the control using end-point control or an equivalent [9,13]. End-point sensing would be very difficult to implement in space, particularly for large manipulator motions. These methods also leave spacecraft vibration virtually uncontrolled, which could present serious hazards in space. The problem of planning manipulator motions to reduce the vibrations in elastically constrained manipulators has yet to be addressed.

In this study just such a method is proposed by exploring the nonlinear dynamic characteristics of an effectively rigid manipulator, such as the SPDM, mounted on a highly flexible supporting structure, such as the SSRMS. The nonlinear nature of the system makes it possible to plan paths taken by the rigid manipulator between given end-points that reduce its dynamic disturbance to the elastic supporting structure and, hence, its vibrations. These paths may also be used in conjunction with filtering techniques to find the velocity profiles along the paths that further reduce vibration levels.

These reduced vibration paths are planned using a technique called the Coupling Map, an analytical tool describing nonlinear dynamic interaction between the manipulator and its elastic base. The Coupling Map evolved from a technique called the Enhanced Disturbance map, or EDM, which has proven effective in understanding the problem of dynamic disturbances to free-flying spacecraft caused by manipulator motions, and in finding paths that reduce such disturbances [7,17].

## 2 Analytical Development

### 2.1 System Dynamic Model

Consider an  $n$  degree-of-freedom (DOF) rigid body manipulator mounted on a flexible base in a zero gravity environment, see Figure 2. The flexible base is modeled as a linear 6 DOF elastic structure. We assume that the distributed mass of the supporting structure is small compared to the mass of the manipulator and its base in Figure 2. For this system we write a set of generalized coordinates as  $\xi = [\phi, q]^T$ ;  $\phi$  represents the 6 generalized coordinates describing the position and orientation of the manipulator's base frame with respect to a Newtonian reference frame. The  $n$  elements vector  $q$  represents the  $n$  manipulator joint displacements. A corresponding generalized force vector is defined as  $\Xi = [\tau_\phi, \tau_q]^T$ , and the system equations of motion are written in the form:

$$H(\xi)\ddot{\xi} + C(\xi, \dot{\xi})\dot{\xi} + K\xi = \Xi \quad (1)$$

where  $H(\xi) \in \mathbb{R}^{n+6 \times n+6}$  is a symmetric, positive-definite inertia matrix,  $C(\xi, \dot{\xi}) \in \mathbb{R}^{n+6}$  is the force/torque vector accounting for centrifugal and Coriolis effects, and:

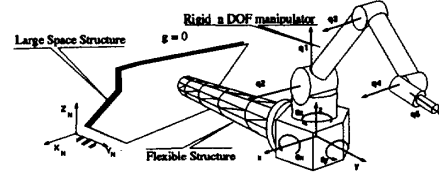


Figure 2: Elastically constrained space manipulator

$$K = \begin{Bmatrix} K_b & 0 \\ 0 & 0 \end{Bmatrix} \in \mathbb{R}^{n+6 \times n+6} \quad (2)$$

is a symmetric matrix representing the stiffness of the system.  $K_b$  is the stiffness matrix of the base. The control torques for the manipulator joints are contained in  $\Xi$ .

We write a generalized momentum vector  $\pi$  for the system based on the selection of generalized coordinates as:

$$H(\xi)\dot{\xi} = \pi \quad (3)$$

where the elements of  $\pi = [\pi_\phi, \pi_q]^T$  correspond to the components of the generalized momentum in the direction of the generalized coordinates. Equation (3) is used to develop an algorithm for computing the Coupling Map discussed in the next section. Equation (1) is the basis of a simulation program, *RiBS*, used to evaluate the performance of the Coupling Map-based path-planning algorithms.

### 2.2 The Coupling Map

The Coupling Map is an analytical tool graphically describing the sensitivity of the transfer of vibrational energy into the system's compliant structure due to manipulator motions. In general, this energy transfer is a complex phenomenon that depends on factors such as the instantaneous direction and velocity of the manipulator motion, the instantaneous motion of the base, the mass properties of the manipulator and the base, the stiffness of the compliant structure and the configuration of the system.

The Coupling Map is developed using the following assumptions to simplify the problem:

1. Gravity forces are negligible.
2. The forces and torques exerted on the manipulator by the structure are small.
3. The disturbances of the manipulator on its structure do not excite structural resonance effects.
4. The manipulator begins its motion from rest.
5. The end-effector does not interact with the environment during the manipulator motion.

Some of the above assumptions, while required for analytical development, will be violated in practice. However, as discussed later, simulations and experimentation show that the technique yields good results even in these cases.

Based on the above assumptions, the total generalized momentum of the manipulator remains small and Equation (3) becomes:

$$H(\xi)\dot{\xi} = \pi = 0 \quad (4)$$

We write the inertia tensor  $H(\xi)$ , based on the choice of generalized coordinates, as:

$$H(\xi) = \begin{Bmatrix} A & B \\ C & D \end{Bmatrix} \quad (5)$$

where  $A \in \mathbb{R}^{6 \times 6}$  is a symmetric submatrix relating the linear and angular velocity vectors of the manipulator base to its linear and angular momenta, and  $B \in \mathbb{R}^{6 \times n}$  is a mass submatrix relating manipulator joint motion to manipulator base linear and angular momenta. Submatrix  $C = B^T$  and submatrix  $D \in \mathbb{R}^{n \times n}$  relates the manipulator joint velocities to their momenta. Using Equation (5) and recalling that  $\dot{\xi} = [\dot{\phi}, \dot{q}]^T$ , we solve Equation (4) for  $\dot{\phi}$ , yielding:

$$\dot{\phi} = -A^{-1}B\dot{q} \quad (6)$$

When we replace the derivative operation by a variation, and let  $G = -A^{-1}B$ , Equation (6) becomes

$$\delta\phi = G\delta q \quad (7)$$

The matrix  $G$  is called the system's disturbance matrix and relates infinitesimal manipulator motion,  $\delta q$ , to infinitesimal base motion,  $\delta\phi$ , for a free-floating system [7,17]. This physical interpretation is not strictly true for elastically constrained systems, since Equation (7) is based on the assumption of zero external forces or torques acting on the system. However, if low-coupling paths are found, then it can be argued that the forces and torques in the directions that are susceptible to large vibrations, those with low stiffness, are small. The quantity  $\delta\phi$  represents a measure of the dynamic coupling between the manipulator and its supporting structure for the systems considered in this study, and is called the instantaneous base dynamic disturbance. For systems that consist of a spacecraft-borne manipulator in which the spacecraft is kept stationary in inertial space, say by spacecraft attitude control reaction jets, the quantity  $\delta\phi$  has been shown to be proportional to the reaction forces/torques required to keep the spacecraft stationary [7,17]. In other words, when manipulator motion results in a relatively low dynamic disturbance,  $\delta\phi$ , low reaction forces/torques are required to restrain the system. Similarly, the force/torque required from the supporting structure to restrain the manipulator base is also proportional to the dynamic disturbance  $\delta\phi$ , or:

$$\tau_\phi \sim \delta\phi \quad (8)$$

for the systems considered in this study, which are constrained by elastic rather than reaction jets.

These forces and torques are clearly equal and opposite to the force and torques acting on the structure, and result in energy being transferred into the elastic supporting structure over a given manipulator motion. The residual vibration excited by manipulator motion is directly related to the magnitude of this energy. So, for a system with *given stiffness characteristics* and a given speed of maneuver, manipulator paths that have low instantaneous disturbances along the path,

$\delta\phi$ , will produce relatively small forces/torques acting on the supporting structure; hence relatively small amounts of energy will be transferred to the structure. Low residual vibrations would be expected as a result.

The energy transferred to the supporting structure will also be a function of the stiffness characteristics of the structure for a given set of disturbance forces and torques. Simple mechanics shows that more strain energy is required in a soft structure to resist a given force than in a stiff structure. Hence, the strain energy that will be produced in a supporting structure is a function of how the structure's effective stiffness matrix aligns with the disturbance force/torque vector. This is modeled as follows.

The strain energy  $V$ , stored in the compliant structure, can be written in terms of this stiffness matrix  $K_b$ , and the force/torque exerted on this structure by the manipulator base, as:

$$V = \frac{1}{2}\tau_\phi^T K_b^{-1}\tau_\phi \quad (9)$$

Equation (8) gives the forces/torques which would act on the structure, assuming the system is at rest with no significant elastic deformation, and the manipulator joints move by a small amount. The strain energy introduced into the structure, then, using Equation (9), is:

$$V \sim \delta\phi^T K_b^{-1}\delta\phi \quad (10)$$

By recalling Equation (7), we can write Equation (10) as:

$$V \sim \delta q^T G^T K_b^{-1}G\delta q \quad (11)$$

A matrix  $Q$ , called the Coupling Matrix, is now defined as  $Q = G^T K_b^{-1}G$ ; Equation (11) is written as:

$$V \sim \delta q^T Q\delta q \quad (12)$$

The Coupling Matrix  $Q$  is a function of the manipulator's configuration and is a measure of the manipulator's sensitivity to the transfer of vibrational energy to its supporting structure. The matrix  $K_b$  enters into the formulation so that directions of base motion with low stiffness carry a higher weight than those with high stiffness.

A singular value decomposition of the Coupling Matrix  $Q$  yields directions and magnitudes of maximum and minimum energy coupling in the configuration space defined by manipulator joint motions,  $q$  [15]. The directions of minimum coupling are used to plot lines of minimum energy in this space similar to the lines of zero disturbance found in the Enhanced Disturbance Map for free-floating systems [7,16,17]. The Coupling Map consists of these minimum energy lines plotted in an  $n$ -dimensional joint space, see Figure 3.

Moving the manipulator along these lines is likely to result in relatively low amounts of strain energy being transferred into the system's elastic supporting structure, with little resulting vibration. Motions across or perpendicular to minimum energy lines are likely to result in a local maximum transfer of energy

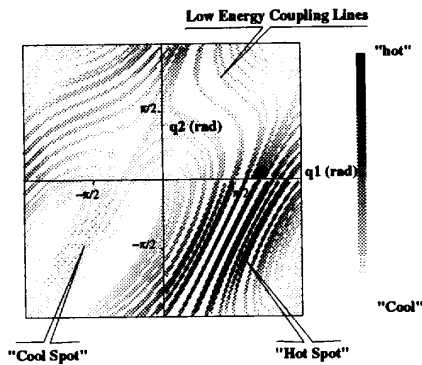


Figure 3: The Coupling Map

to the system's elastic supporting structure, resulting in large residual vibration. The relative coupling strength is shown on the Coupling Map by making the lines darker when the strength is high ("hot" areas) and lighter when the strength is low ("cool" areas).

### 3 Elastically Constrained Systems and the Coupling Map

The formulation of the Coupling Map is based on several assumptions which are not strictly valid for a system which is experiencing significant base motion. However, as the structural disturbances produced by the manipulator motion approaches zero, the instantaneous directions of minimum and maximum coupling predicted by the Coupling Map become valid. Hence, low coupling paths found using the Coupling Map should be close to ones of minimum excitation. These paths can be used either directly or as initial conditions in more computationally intensive optimization procedures to reduce computation times and enhance the likelihood that these procedures will converge to good solutions. Several algorithms for finding low excitation paths exist, based on the understanding provided by the Coupling Map [16]. Here we present one of these algorithms, the *Hot Spot* method. Simulation results show that it finds paths that result in a reduction of the amplitude of the residual vibrations of the manipulator base after a manipulator maneuver.

#### 3.1 Example 1: A Simple System

Consider the system in Figure 2 and its Coupling Map depicted in Figure 4. The properties of this system are similar to the dynamic characteristics of an experimental manipulator called the ARM II, currently located at NASA's Langley Research Center.

The system in Figure 2 is mounted at the end of a flexible structure with elastic properties similar to those of the Space Shuttle Remote Manipulator System (RMS). The system's first vibrational mode frequency is between 0.4 Hz and 0.7 Hz, depending on the ARM II's configuration. It is assumed to have a structural damping ratio of 0.05.

The system must move the manipulator's end-effector from an initial point *a* to a final position point *b* in inertial space. The initial and final configura-

tions in the joint space Coupling Map corresponding to these points are shown in Figure 4. As described, the dark areas in the Coupling Map represent hot spots, areas in which moving perpendicular to the lines results in high maximum energy transfers to the base compliant structure. Notice that the final configuration, point *b* in this example, lies in a hot spot. In this case, motions of the manipulator joints perpendicular to the minimum coupling lines should produce a substantial energy transfer to the base's softer modes. Thus, the hot spot approach is as follows:

*If a path must go through Coupling Map regions of large energy coupling, then it should follow minimum coupling lines as closely as possible. When coupling is low, at a cool spot, the path may move across the minimum coupling lines.*

Three paths are considered to demonstrate this approach, shown in Figure 4. The first path, *path1*, is a straight line from point *a* to *b* in joint space. The second, *path2*, was chosen with the hot spot method. Finally, *path3*, apparently an appropriate one, moves perpendicular to minimum coupling lines in the hot spot. The three maneuvers were designed to take approximately 2 seconds each.

The dynamic response of the system, including its supporting structure, was computed using our *RiBS* software. *RiBS* is a mobile base dynamic simulation package developed as part of this research. The time history of the vertical motion of the manipulator base (*z*, shown in Figure 2) for the three maneuvers is shown in Figure 5. Notice that *path2*, the path computed using the hot spot method, yielded the lowest vibration amplitude. The *path3* produced the highest residual vibration, agreeing with the predictions made by the Coupling Map. This path crosses the minimum coupling lines in a hot spot region. Figure 5 also shows that, given a base motion tolerance of 0.02m, *path2* resulted in the lowest settling time of 10.5 seconds. The *path1*, a straight line path, produced a base vibration that took more than twice as long to settle. This increased settling time can dramatically increase the cost of space missions; currently the Space Shuttle costs approximately \$20,000 per minute to operate [14]. Therefore, for this one simple maneuver, using *path2* would have save \$3,300 over the more conven-

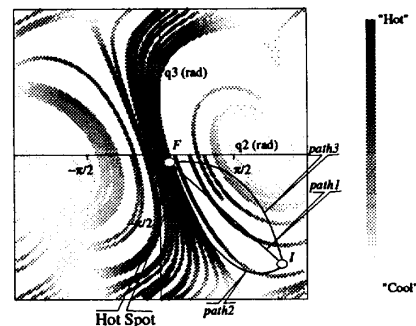


Figure 4: Coupling Map for link 2 and 3 of the NASA's ARM II mounted at the end of the RMS.

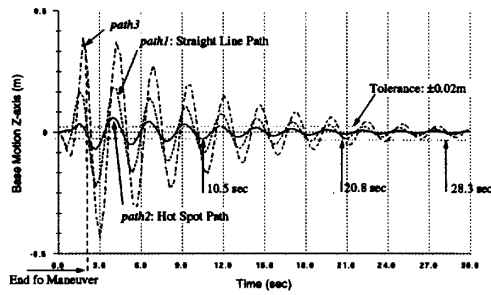


Figure 5: Residual vibrations in the base Z-axis for *path1*, *path2* and *path3*.

tional straight line path.

### 3.2 Example 2: A more Complicated System

A prototype of a compliant base space manipulator system was designed in some detail to test the effectiveness of this method on more realistic systems. This prototype, shown in Figure 6, is based on the SPDM mounted at the end-effector of the SSRMS [8], see Figure 1. Our dextrous manipulator system has 19 DOF: two 7-DOF arms connected to a 5-DOF body. A detailed description of the prototype is found in [16]. This system has a total mass of 455 Kg. The 19 DOF system is mounted at the end of a 50 ft. long flexible structure with stiffness characteristics similar to those of a massless Space Shuttle RMS with a structural damping of 5%. Its effective stiffness matrix,  $K_b$ , (in  $N/m$  for translational motion and  $Nm/rad$  for rotational motion) is:

$$K_b = \begin{Bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 30.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 9.8 & -7.6 & 0.0 & 0.0 \\ 0.0 & 0.0 & -7.6 & 79.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 15.4 & 0.0 \\ 7.6 & 0.0 & 0.0 & 0.0 & 0.0 & 79.5 \end{Bmatrix} \times 10^4 \quad (13)$$

The system's task considered here consists of two steps. The first is to reposition its 5 DOF body so as to bring its two 7 DOF arms and camera system closer to a working station. The second is to move one of the 7 DOF arms from an initial to a final configuration. These steps are planned using a simple straight line path and the Hot Spot method.

The first step requires moving three of the system's five body joints from an initial to a final configuration while keeping the two 7 DOF arms stationary. The initial and final configuration of the prototype body is shown in Figure 7. This three-joint system needs a three-dimensional Coupling Map. However, a full graphical representation of the Coupling Map is not necessary to compute the hot spot-based path. For this example only the minimum coupling lines passing through the initial and final configuration, points *a* and *b*, are computed, see Figure 7. Figure 7 shows that point *b* lies on a "hotter" region than point *a*.

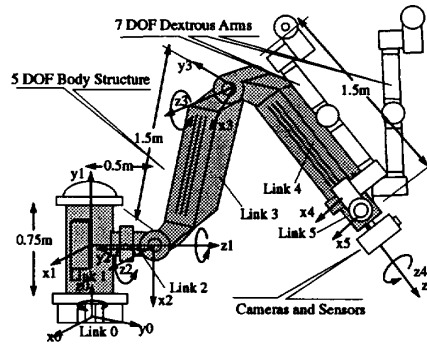


Figure 6: The Prototype System.

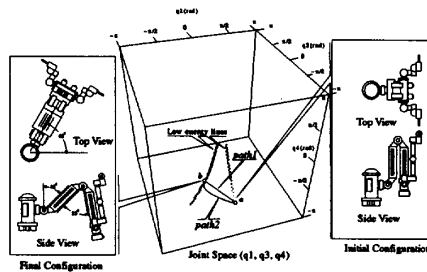


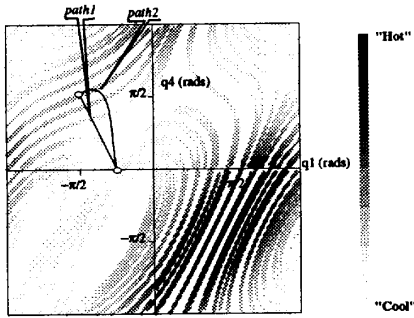
Figure 7: Computed paths for three joints on the prototype body.

This suggests that a low coupling path can be found by starting perpendicular to the minimum coupling line at point *a* and approaching point *b* parallel to the minimum coupling line at *b*. This path is shown as *path2* in Figure 7 and takes 3 seconds. A straight line path, *path1*, in Figure 7 is used for comparison. The simulation results show that *path2* results in a 14.3% reduction in the magnitude of the residual vibration based on the system's base absolute motion when compared to *path1* for the same maneuver time.

The second step of the task requires two of the seven joints on the system's right arm to move from an initial to a final configuration while keeping the body stationary at the final configuration described above. The motion is planned to take 2.0 seconds. A Coupling Map is computed and two paths generated and simulated. Figure 8 shows the Coupling Map for joints 1 and 4 of the prototype's right arm, assuming that the rest of the joints of the system are kept stationary. The simulation shows that the residual vibrations produced by the straight line path takes 4.4 seconds to die out to an amplitude of less than 2 mm. The residual vibrations produced by the "hot spot" path only takes 3.4 seconds to reach this level.

## 4 Conclusion

We have developed a technique called the Coupling Map to aid both in understanding the problem of vibrations in a space manipulator carried by a highly flexible structure, such as another large manipulator,



**Figure 8: Coupling Map for joint 1 and 4 SPDM right arm and computed paths.**

and to plan paths which reduce these vibrations. Simulation results suggest that this technique produces useful answers even though a number of simplifying assumptions were made in the development of this tool. These paths can be used directly to plan the motion of flexible mounted space manipulators, or used as initial values in more computationally intensive numerical optimization procedures to improve their convergence. While the graphical construction of the Coupling Map for higher degree of freedom systems is not possible, the algorithms suggested by this technique when applied to lower DOF systems may be extended to higher order systems to enable computers effectively to search through the higher order configuration space for reduced excitation paths. An experimental program is currently in progress at MIT. Preliminary experimental results obtained have shown the practical effectiveness of the method. A detailed description of these results is beyond the scope of this paper.

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