PLANNING MOTIONS OF ROBOTIC SYSTEMS SUBJECT TO FORCE AND FRICTION CONSTRAINTS WITH AN APPLICATION TO A ROBOTIC CLIMBER

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ABSTRACT

Multi-limb robots able to apply large forces over large ranges of motion are needed for many tasks. These tasks may strain a system's actuation capabilities and its ability to maintain secure contact with its environment. The *Force-Workspace Approach* is presented and used to plan a system's activities without violating actuation limits or frictional constraints with their tasks and environments. The technique is applied to the example of a robotic climbing machine.

1. INTRODUCTION

Mobile, multi-limb systems have been proposed to perform hazardous missions such as space construction and repair, planetary exploration, and toxic-waste clean-up [1, 2]. These systems will need to move through large ranges of motions while applying large forces. Unlike conventional industrial robots, mobile systems will not be firmly attached to a factory floor and can lose handholds or footings, or overturn. In addition, their actuators will be limited by their need to be lightweight and power efficient, particularly for space systems. Planning motions to enable a system to apply large static forces while moving through large ranges of motion subject to actuator constraints, contact force and moment constraints, and kinematic constraints is an important problem that has yet to be solved.

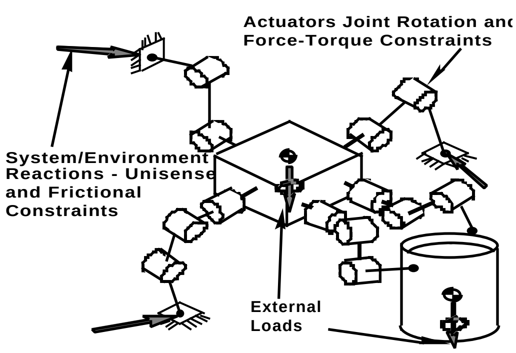


Fig. 1. A Mobile Robotic System Concept

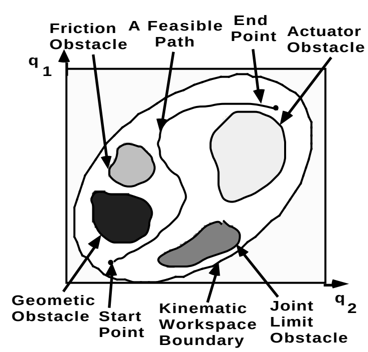


Fig. 2. The Force-Workspace Concept

A multi-limb robotic system and its environment generally form redundantly actuated open and closed kinematic chains, see Fig. 1. The actuator efforts and contact wrenches for a given task are indeterminate. A number of methods to specify these have been developed, primarily within the context of grasping in robotic hands and walking robots [3-8]. For example, methods have been developed to solve for contact forces and actuator efforts subject to both frictional contact constraints and actuator effort limits [5]. Some were done in the context of robotic walking machines [6]. While most of these studies have focused on systems in fixed configurations, some have treated systems with *pre-specified* motions. For example, methods to find smooth actuator efforts and contact wrenches along pre-specified system motions have been proposed [6,7].

These works provide important tools. However, the question of how to plan a system's motions so that it is able to perform tasks requiring large forces over substantial ranges of motion remains to be addressed; for example: how can a robotic device lift a heavy object in its "arms" while standing or climbing on a soft sandy hill?

In this paper a method called the *Force-Workspace* (*FW*) *Approach* is presented to generate motions for a mobile, multi-limb system which will allow it to handle a given set of loads over large ranges of motion without saturating its actuators, violating contact constraints between the system and its load or environment, and the constraints imposed by its joint motions and kinematic structure. In the *FW* method these constraints are mapped into the system's configuration space (C-space) to form *constraint obstacles* similar to geometric C-space obstacles [8-10]. Thus the constraints imposed by the system's force capabilities, by its kinematics and by obstacles in the environment can all be considered in a uniform way when the system motions are planned. Fig. 2 shows schematically a two-dimensional C-space, [q_1, q_2]. Within the kinematic workspace boundary, the constraints due to contact conditions, actuator saturation limits, joint limits, and environment obstacles are all mapped as C-space obstacles. To plan feasible system motions a path is chosen in C-space which does not intersect any of these obstacles.

To demonstrate this method, it is applied to plan the motions of a three-limb, planar, articulated climbing machine given the task of climbing upwards between two vertical walls, pushing outwards to maintain frictional support without saturating its actuators. It is shown that it is not able to achieve this feat for all paths. The *FW* is then used to generate feasible gaits for the system to climb successfully.

2. GENERATING THE FORCE WORKSPACE (FW)

Systems studied are assumed to have rigid links and ideal joints. They are comprised of a main body with M limbs which contact the environment or task; the contacts may be with a kinematically constrained object such as a valve handle. Externally applied forces and moments are assumed to be functions of its configuration, and may act at any location on its structure. Such a system forms a mechanism with n degrees of motion freedom (DOF). The system's configuration space (C-space) is parameterized by a n×1 vector $\mathbf{q}$ and a pose parameter P. To generate a system's *FW* the C-space is represented by a recursive data structure called a generalized quadtree or 2^n-tree [11,12]. The nodes of the tree represent cells in an n dimensional space, and the root node represents the entire space under investigation. To generate the 2^n-tree, every configuration within a node is tested to determine if it is *feasible*, *infeasible*, or *mixed*, with respect to system constraints. At all configurations within a feasible node no constraints are violated. At all configurations within an infeasible cell at least one constraint is violated. If a node contains both feasible and infeasible configurations, the node is labelled mixed. The problem of mapping system constraints as obstacles into the C-space to generate the *FW* is reduced to generating the appropriate feasibility tests corresponding to sets of system constraints.

Three tests are developed. The first is used to determine the kinematic workspace of the multi-limb system [13,14]. The second test is used to map joint limits and workspace obstacles as obstacles in the C-space [13,14]. The third test is used to map actuator saturation limits and contact constraints as obstacles into the C-space. This latter test is performed by first determining if a single point in the C-space is feasible. This is done by extending a well-known linear programming technique, originally developed for robotic hands [5]. Unisense contact constraints (such as when limbs can push when contacting objects, but cannot pull), linearized coulomb friction constraints, and actuator effort limits can be written as linear inequality constraints on the intensities of contact wrenches exerted between the sysem and its environment [5]. These linear inequality constraints can be mapped into a space of "internal forces" in the system. If the largest possible circle, or in general a hypersphere, is inscribed within the constraint polygon, its center will be a maximum distance from the nearest constraint planes, and its radius, termed d^*, will be this distance. These can be solved for via a modification of the linear programming approach presented in [5], as shown in [13].

In this study, this technique is used to determine the feasibility of a configuration by noting that if d^* is greater than zero, a configuration will be feasible, and if d^* is less than zero, it will be infeasible. This can be extended to determine if an entire C-space node (a cell within the C-space) is feasible, infeasible or mixed by determining if at *every* configuration within the cell, d^* is greater than zero, less than zero, or if at some configurations within the cell it is greater than zero and at others it is less than zero. This is done using a nonlinear programming technique described in detail in reference [13].

3. MOTION PLANNING IN THE FORCE-WORKSPACE

To plan the motions of a multi-limb mobile robotic system applying large forces, paths must be found in the C-space which avoid constraint obstacles. To automatically generate such paths, the feasible nodes of the 2^n-tree representation of the force-workspace are transformed into a search graph whose edges represent the physical adjacency relationships between all feasible cells in the force-workspace. Weighting the edges of the search graph with a configuration based performance criteria and using a minimum cost graph search allows paths to be planned and tailored to suit particular applications.

The above procedure allows motions to be planned within a single force-workspace, that is, for a given *stance*. A stance is defined as the system making a particular set of fixed contact locations with its environment or task. In order to relocate contacts, for example while making a step with a walking machine or while turning a valve "hand-over-hand," a method is required to move between force-workspaces. In other words, each time a new set of contact locations is chosen, the system forms a new mechanism, or *resides in a new stance*, and a new force-workspace will be generated to plan motions for this mechanism. During the process of transferring between stances, or equivalently between force-workspaces, it can be shown that system constraints will not be violated if the system resides in an intersection of feasible regions within the force-workspaces corresponding to each stance [13]. A complete description of this application can be found in reference [14].

4. AN APPLICATION—A PLANAR CLIMBING ROBOT

Fig. 3 shows a planar, three-limb, robot which must climb between two walls by pushing outwards against them in order to generate frictional support. Its parameters are given in Table 1. The limbs are assumed weightless compared to the body and payload. The contacts made with the walls are point contacts with friction which do not support moments between the limb tips and the walls, but which support coulomb friction forces of the form $F_T \leq \mu F_N$ where F_N is a normal contact force and F_T is a tangential contact force as shown in the figure.

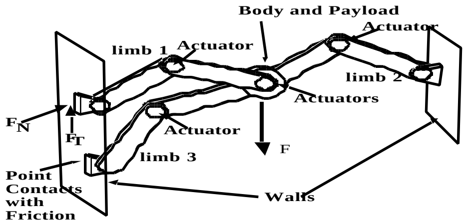


Fig. 3. A Planar Climbing Robot

Table 1. System Parameters

$\tau_{1,3\ max/min}$	±200 Nm	Joint torque limits
$\tau_{2\ max/min}$	±300 Nm	Joint torque limits
a, b	0.5 m	Link lengths
d	1.0 m	Wall separation
h	0.4 m	Step height -See Fig. 4
μ	0.8	Coefficient of friction
$\|\mathbf{F}\|$	300 N	Task wrench, zero pitch

The *FW* approach can be used to find motions that will allow the system to climb. Not all system motions will succeed since, in certain configurations, the system will violate one or more of its friction or actuator constraints. It can be shown that an acceptable gait is achieved as follows. First, the system uses limb 1 and limb 2 to lift its body upwards; this is referred to as a two-limb pull-up. Then, the third limb, limb 3, is placed at a new contact location and the supporting loads are transferred to limb 3, allowing limb 1 to be lifted from the wall. The payload can then continue being lifted using a new pair of limbs, limb 3 and 2, and the process repeated. The key to the solution is to use the *FW* approach to find acceptable two limb motions. The system C-space used for this system *FW* is defined as all $\{\mathbf{q}, P\}$ where the two-limb system can move given fixed contact locations on each wall, see Fig. 4.

The configuration space of the system is parameterized by $\{\mathbf{q} = [\theta_1, \theta_2]^T, P = 1,2\}$. Fig. 5 shows the *FW* for $P = 1$(right limb elbow-down configuration), and for a system with parameters given in Table 1. Unfortunately, space limitations do not permit a presentation of the details of this analysis and the reader is referred to references [13,14].

The dark grey cells represent configurations outside the kinematic workspace (KW), the medium grey cells are constraint obstacles within the KW where either actuator torques or wall frictional constraints are violated, and the light grey cells represent feasible configurations where the system may travel. Joint limit obstacles are not shown. The white cells are mixed cells. Note that a *merge* operation was performed on the final map after the subdivision process, explaining why there are white cells which are larger than the minimum cell size. The FW can be mapped into the coordinates of the center of the body or X–Y space, see Fig. 6a. The figure shows a single X–Y space map for $P = 1$.

To plan a two-limb pull-up using the *FW*, finely divided feasible nodes are used to generate a weighted search graph. Fig. 6a and Fig. 6b. show a resulting minimum distance path travelled by the center body between points A and B, with the added constraint that they remain a minimum distance from the constraint obstacle. This path avoids the large constraint obstacle in the left of the FW which can be shown to be due to the actuator 1 torque limit, and the left wall friction constraint [13]. It can be shown that while the system does not violate either of the constraints, it comes quite close to them as the path moves close to the constraint obstacle [14].

Other weighting criteria can be easily selected to keep these values away from their limits[13]. It has also been shown that it is possible to plan an effective climbing gait for this system using the FW approach [13].

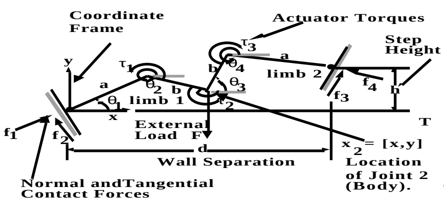


Fig. 4
Two limb subsystem of the climbing robot

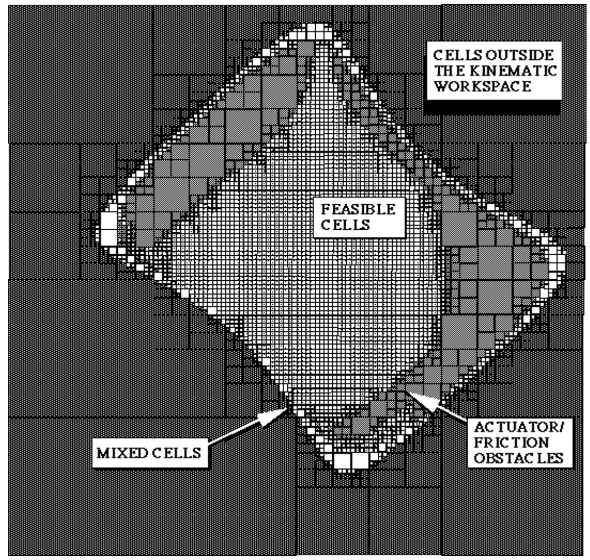
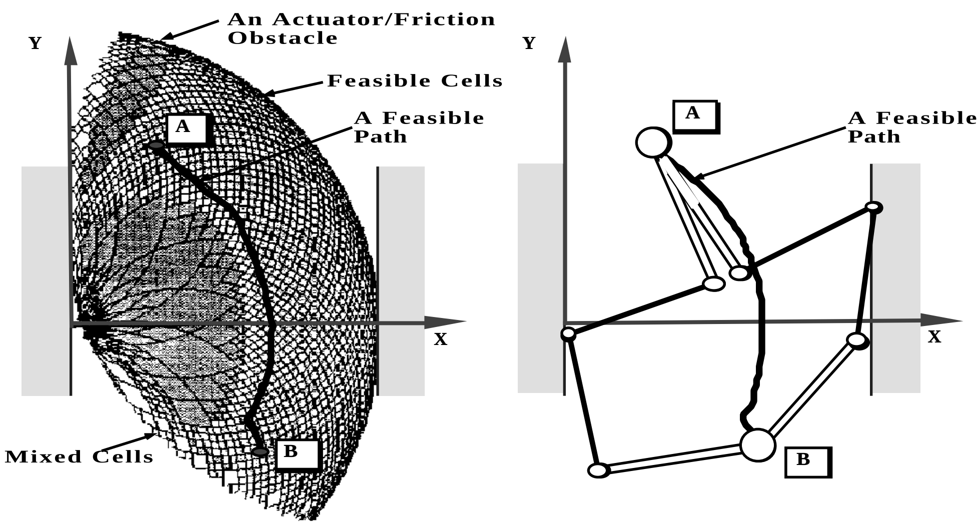


Fig. 5 The force-workspace in $[\theta_1, \theta_2]$–space



a. A FW Path in Body Center Coordinates **b. Body Motions on Path**

Fig. 6 Path Planning Using the Force-Workspace Approach

5. CONCLUSIONS

A method, termed the *Force-Workspace* (*FW*) method, has been presented to generate motions for multi-limb robotic systems enabling them to apply large forces over large ranges of motion without saturating actuator effort limits, system-environment friction constraints, or kinematic joint limits. The *FW* method maps these constraints into the system C-space to form *constraint obstacles*. To generate motions along which actuator efforts can be specified without violating system constraints, paths are planned that avoid these constraint obstacles. The method is applied to a three-limb robot whose task is to climb between two vertical walls by pushing outwards to generate frictional support. Motions were planned automatically within the system force-workspace enabling it to lift itself upwards using two limbs at a time, as required by a proposed climbing gait

6. ACKNOWLEDGEMENTS

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