A New Design Approach for Solar Concentrating Parabolic Dish Based on Optimized Flexible Petals

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Abstract

Large parabolic dish concentrator mirrors are an important component of many solar energy systems. They need to be relatively precise and are expensive to fabricate and to transport. Here, a new concept for designing and fabricating large parabolic dish mirrors is presented. The dish mirror is formed from several optimal-shaped thin flat metal petals with highly reflective surfaces. Attached to the rear surface of the mirror petals are several thin layers whose shapes are optimized to have reflective petals form into a parabola when their ends are pulled toward each other by cables or rods. An analytical model to optimize the shape and thickness of the petals is presented. The validity of the concept is demonstrated using Finite Element Analysis and laboratory experiments. The concept would permit flat mirror elements to be easily fabricated and efficiently packaged for shipping to field sites where they can be assembled into the parabolic dish concentrators. The concept has the potential to provide precision solar parabolic solar collectors at a substantially lower cost than conventional methods.

Keywords: Solar Collector Design, Parabolic Dish Mirrors, Low Cost Fabrication.

1. Introduction

1.1 Background

Large solar mirror collectors are a major subsystem of many solar energy systems, particularly for solar thermal generators. Thermal systems may use many collectors covering large sites [1]. Parabolic dish concentrators offer the highest thermal and optical efficiencies of all the current concentrator options [2]. An analysis of normalized dish cost per unit aperture area as a function of dish radius has been presented previously and

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supports a choice of most cost-effective size between 400 and 1000 m² [3]. See Figure 1.

Figure 1. A 500 m² Paraboloidal Dish Solar Concentrator Developed by Australian National University [2]

Parabolic dish systems consist of a parabolic-shaped dish concentrator that reflects solar radiation onto a receiver mounted at the focal point. These concentrators are mounted on active tracking systems to follow the sun. The heat collected by the receiver is typically used by a heat engine mounted on the receiver that moves with the dish structure, such as Stirling and Brayton cycle engines. To be most effective the parabolic-shaped concentrator needs to focus the sunlight on the receiver and hence the shape of the parabola needs to be relatively precise.

Current concentrators are usually fabricated using conventional structures that made them expensive [2, 4-11]. New methods are required to reduce these costs, such as the concept presented here.

1.2 Prior work

The concept for parabolic dishes is an extension of our previous work on optimized solar parabolic trough collectors, using elastic bands [11] [12]. The band shape is optimized by varying its width so that it forms a parabola when its ends are pulled together to a known distance, see Figure 2. By varying the width of the band, the bending stiffness of the band becomes a function of the length long the band. Optimally selecting this function results in the band achieves a parabolic shape when it is bent. A set of bands can be used to form a backbone, or supporting structure for the trough mirror by mounting a flexible flat mirror on them. The bands have in general a uniform thickness and can be easily and inexpensively stamped from sheet steel and shipped in flat packages to a site. At the site, its ends would be pulled together to a given distance by a wire, or rod, or actively controlled with a simple control system. The backbone band was experimentally found to be effective.
As discussed below it would be difficult to directly apply the variable width concept to a parabolic dish collector.

1.3 Approach and Summary

Here it is shown that a dish mirror is formed from several optimal-shaped thin flat metal petals with a highly reflective surface with the optimal bending stiffness achieved by varying the petal’s thickness (instead of its width) as function of its length, since the final result cannot have any gap between petals. Varying the petal’s thickness continually is difficult and expensive. However it is approximated by building up the petal using several layers whose figures are optimized so when the petal is bent it will form a parabola. Note that each of the pedals will have a single curvature, such as shown in Figure 3. The double curvature problem is beyond the scope of this paper.

An analytical model to optimize the petal thickness is presented. The concept would permit flat mirror elements to be easily fabricated and efficiently packaged, shipped and assembled on site. The fundamental validity of the concept is demonstrated using Finite Element Analysis and experimental data.

### Figure 3. Parabolic Dish Parameters and Variables

#### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>b(x), b(s)</td>
<td>Petal width as functions of x or s</td>
</tr>
<tr>
<td>D</td>
<td>Dish diameter</td>
</tr>
</tbody>
</table>
\[ d = \text{Dish depth} \]
\[ d_F = \text{Focal area diameter} \]
\[ F = \text{Load} \]
\[ f = \text{Focal length} \]
\[ N = \text{Number of petals} \]
\[ s = \text{Parabolic arc length} \]
\[ x, y, z, X, Y, Z = \text{Coordinates} \]
\[ \theta = \text{Rim angle} \]

The equation of a paraboloid can be written as:

\[
z(x, y) = \frac{x^2 + y^2}{4f} \quad (1)
\]

The parabolic arc length \( s \) is given by as a function of \( x \):

\[
s(x) = \int_0^x \sqrt{1 + \left(\frac{u}{2f}\right)^2} \, du \quad (2)
\]

where \( u \) is a dummy integration variable along the longitudinal direction of the petal.

Hence,

\[
x = 2f \cdot \ln(s / 2f + \sqrt{s^2 / 4f^2 + 1}) \quad (3)
\]

If the dish consists of \( N \) petals, which only have the parabolic curve along the long arc as discussed above, the width of the petal can be obtained as a function of \( x \) is:

\[
b(x) = 2x \cos\left(\frac{\pi}{N}\right) \quad (4)
\]

Hence

\[
b(s) = 4f \cdot \ln(s / 2f + \sqrt{s^2 / 4f^2 + 1}) \cdot \cos\left(\frac{\pi}{N}\right) \quad (5)
\]

The resulting shapes for the petals of a 44 pedal dish shown in its flat state (prior to bending) -as it might look after fabrication are shown in Figure 4. It should be noted that the sides of the pedal are not precisely straight lines, although when the dish is composed of many petals they will appear straight. The number of petals is a design choice. As will be shown below, the more petals, the better the single curvature design will perform in terms of the smaller the receiver to collect all the sunlight. However, the larger number of
petals, the more complex the fabrication and the larger the number of cables required.

2.2 Optimal Petal Thickness as Function of $s$
Assuming Euler-Bernoulli beam theory applies, the deflection of the petals is determined by [13]

\[ M(s) = EI(s) \frac{d\phi(s)}{ds} = EI(s)\kappa(s) \]  

where $M(s)$ is the bending moment on the band, $\delta\phi$ is the rotation of the band surface normal, and $\kappa(s)$ is the curvature of the final band shape, as shown in Figure 5, with $I(s)$ and $\kappa(s)$ for the petal given by:

\[ I(s) = \frac{M(s)}{E\kappa(s)} \]  

\[ \kappa(s) = \frac{1}{2f} \left(1 + \ln^2 \left(\frac{s/2f + \sqrt{s^2/4f^2 + 1}}{s/2f - \sqrt{s^2/4f^2 + 1}}\right)\right)^{\frac{3}{2}} \]

it follows
where \( t(s) \) and \( b(s) \) are the petal thickness and width as functions of \( s \), and \( h_0 \) is the load position, as shown in Figure 5.

\[
M(x) = F \cdot \left( h_0 + d - \frac{x^2}{4f} \right)
\]

\[
t(s) = \sqrt[3]{\frac{M(s)}{b(s)E\kappa(s)}}
= \sqrt[3]{\frac{12F(h_0 + d - f \ln^2 (s/2f + s^2 / 4f^2 + 1))}{b(s)E\kappa(s)}}
\]

**Figure 5. Petal Curvature**

### 2.3 Layers’ Shapes

As discussed above, varying thickness as functions of \( s \) can easily be approximated by layers of constant thickness and varying \( b(s) \) based on same stiffness. An example of this is discussed later in this paper.

### 3. Energy Efficiency

In order to optimize the petal designs, a performance metric is established. Here, the metric based on the fraction of energy that enters the dish that is captured by a given size receiver is used. It assumes that the dish tracks the sun; hence the sunlight enters along parallel lines to the dish's ideal paraboloid’s major axis \( (z) \).

Figure 6 shows a petal projected on \( X-Y \) plane. Recall that \( d_F \) is the focal diameter. All the light falling on the shaded area will be captured by the receiver. The fraction of the energy falling on this petal that will be reflected to the receiver can be calculated for petals that have ideal parabolic cross section from elementary geometry as follows.
The shaded effective area ($A_E$) is given by:

$$A_E = \left(\frac{l}{2} + L\right) \cdot d_f = \left(R - \frac{l}{2}\right) \cdot d_f$$  \hspace{1cm} (11)

![Image of a dish antenna with shaded area]

Figure 6. Energy Effective Area

Defining the energy efficiency ($\eta$) as discussed above:

$$\phi = \frac{A_E}{A}$$  \hspace{1cm} (12)

the energy efficiency becomes:

$$\eta = \begin{cases} 
\frac{N}{\pi} \cdot \left(\frac{2d_f}{D} - \frac{d_f^2}{D^2} \cdot \cot(\frac{\pi}{N})\right) & \text{when } \frac{d_f}{D} < \sin(\frac{\pi}{N}) \\
1 & \text{when } \frac{d_f}{D} \geq \sin(\frac{\pi}{N}) 
\end{cases}$$  \hspace{1cm} (13)

The energy efficiency for the dish described in Equation (13) as a function of the number of petals is shown in Figure 7. As expected, for this dish and receiver the efficiency increases as the number of petals increases and as the size of the receiver increases compared to the dish diameter. Keeping a smaller receiver is usually desired as large receivers will obscure more of the dishes aperture, increase the structural requirements of its supporting boom and also increase its undesirable heat-loss to the ambient surroundings.
Figure 7. Energy Efficiency as Function of the Number of Petal Segments

3.1 Focal Error and Actual Energy Efficiency
If the cross section of the petal is not an exact parabola, such as when the thickness of the petal is composed of discrete layers as opposed to the ideal continuous profile, given by Equation (10), the energy efficiency is calculated by ray tracing. When a light ray that is parallel to the dish major axis falls on the mirror dish at a given point, it will ideally be reflected by the mirror so that it passes through the focal point of the parabolic mirror where the receiver is positioned. If the parabola is not ideal, the ray will miss the focal point. If the error is larger than the size of the receiver, its energy is lost and this reduces the mirror’s efficiency. Below a method to calculate the distance of a reflected light ray from a given size receiver is presented. In this analysis the receiver is assumed to be spherical with a diameter of \( d_f \).

Referring to Figure 8 and using the coordinates defined in Figure 3, a light vector along the parabola major axis will be given by: \( \mathbf{L} = (0, 0, -1) \) and the surface normal at arbitrary point \( B (x, y, z) \) is: \( \mathbf{N} = (l, m, n) \). Expression for the angle of reflection, \( \beta \), can be written as:

\[
\cos \beta = \frac{-\mathbf{L} \cdot \mathbf{N}}{||\mathbf{L}|| \\ ||\mathbf{N}||} \quad \sin \beta = \sqrt{1 - n^2}
\]

\[
\mathbf{A} = \frac{-\mathbf{L} \times \mathbf{N}}{\sin \beta} = (-m,l,0)
\]

where \( \mathbf{A} \) is the normal vector to the plane spanned by \( \mathbf{L} \) and \( \mathbf{N} \).
The reflected ray is:

\[
R = N \cdot \cos \beta + (A \times N) \cdot \sin \beta + A \cdot (A \cdot N) \cdot (1 - \cos \beta) \\
= (l \cdot n \cdot (1 + \sqrt{1 - n^2}), m \cdot n \cdot (1 + \sqrt{1 - n^2}), n^2 - (1 - n^2) \cdot \sqrt{1 - n^2})
\]  

(16)

It can be shown that the focal error (the distance that the reflected ray misses the focal point) is:

\[
\Phi = \frac{((0, 0, f) - (x, y, z)) \times R}{|R|}
\]  

(17)

This can be evaluated given a model of the petal shape.

As discussed above, if the focal error is equal to or smaller than focal area radius, \(|\Phi| \leq d_f / 2\), reflected ray will be absorbed by receiver. Otherwise, the rays will miss the receiver.

3.2 Numerical Optimization of Dish Designs

Since it is not likely that the petals of the dish will be simple elements with continuously varying thickness, a numerical optimization method has been developed. In this method a FEM structural model of a petal design is combined with the above ray tracing optimized to maximize the energy efficiency of a petal design by having it match the ideal \(I(s)\) corresponding to the function given by Equations (7) to (10).

4. A Study Case

In this case study, the objective was to find the shape of the layers given the desired design parameters shown in Table 1. Based on the required energy efficiency, \(\eta\), of 90\% or more, and Figure 7, the number of petals \((N)\) was chosen to be 44. For purposes of this example, the material chosen was 2 mm sheet Spring Steel (1095) with a Young’s modulus of 210 GPa and a yield strength of 413 MPa, and Poisson’s ratio is 0.3.
Table 1: Desired Dish Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter $D$ (m)</td>
<td>30</td>
</tr>
<tr>
<td>Rim angle $\theta$ (degree)</td>
<td>180</td>
</tr>
<tr>
<td>Focal length $f$ (m)</td>
<td>18.107</td>
</tr>
<tr>
<td>Dish depth $d$ (m)</td>
<td>3.107</td>
</tr>
<tr>
<td>Focal diameter $d_f$ (m)</td>
<td>1.5</td>
</tr>
<tr>
<td>Energy efficiency</td>
<td>90%</td>
</tr>
</tbody>
</table>

The ideal solution for the petal to meet the design objectives is shown in Figure 9. The cable load required to bend this petal is a reasonable 60 N.

Figure 9. Continuous Petal thickness

Figure 10 shows the ray tracing results for a petal modeled by FEA that does not use the methodology presented here. The petal has a uniform thickness. As can be seen the petal does not have a parabolic shape and most of the light striking the mirror does not fall on the receiver making the concentrating ineffective. Figure 11 shows the mirror with the ideal thickness variation. As shown, all the light falls on the receiver.
Figure 10. Ray Tracing of the Uniform-thickness Petal

Figure 11. Ray Tracing of the Analytical Varying-thickness Petal

Figure 12 shows the layer version of this design. Here each of the layers of the petal is shown separately and then the composite is shown. The layers’ shapes are defined as:
where \( t_0 \) is the constant thickness of the layers; \( b_i(s) \) is the width of the \( i \)th layer, and the varying thickness of the \( i \)th layer before reshaping is defined as:

\[
b_i(s) = \frac{b(s) \cdot t_i^3(s)}{t_0^3}
\]

For this study case, \( t_0=2 \text{ mm} \), and six layers are set to simulate the continuous thickness, so \( i = 1,2,\ldots,6 \).

The layers need to be firmly attached to each other. As discussed in the experimental section, epoxy or spot welding has both shown to be effective for the attachment.

(a) The First Layer

(b) The Second Layer

(c) The Third Layer

(d) The Fourth Layer
Figure 12. A Layered Petal Design

Figure 13 shows the FEA results of this layered petal. Figure 14 shows the ray tracing of the result for the six-layer design. It can be seen that the results provide a good approximation to the ideal continuously varying thickness design, while being far easier and less costly to fabricate.

Figure 13. FEA Results of a Layered Petal
5. Experimental Study

A four-layer design was developed to experimentally validate the design methodology. Its design is shown in Table 2. A 12-petal design was chosen because of its small size and to ease of experimental fabrication. Referring to Figure 7, the expected efficiency is lower than the much larger case study design that was for a 30-meter dish with 44 petals.

Table 2. Experimental System Design Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter $D$ (mm)</td>
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</tr>
<tr>
<td>Rim angle $\theta$ (degree)</td>
<td>90</td>
</tr>
<tr>
<td>Focal length $f$ (mm)</td>
<td>338.0</td>
</tr>
<tr>
<td>Dish depth $d$ (mm)</td>
<td>58</td>
</tr>
<tr>
<td>Focal diameter $d_{f}$ (mm)</td>
<td>56</td>
</tr>
<tr>
<td>Energy efficiency</td>
<td>62.5%</td>
</tr>
<tr>
<td>Petal numbers $N$</td>
<td>12</td>
</tr>
<tr>
<td>Horizontal load $F$ (N)</td>
<td>120</td>
</tr>
<tr>
<td>Material</td>
<td>Spring steel 1095</td>
</tr>
<tr>
<td>Young’s modulus $E$ (GPa)</td>
<td>210</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Yield strength (MPa)</td>
<td>413</td>
</tr>
<tr>
<td>Material thickness $t_{0}$ (mm)</td>
<td>0.8128</td>
</tr>
</tbody>
</table>

Figure 15 shows a top view of the first layer (mirrored) of the 12-petal design. Figure 16
shows the supporting second, third and fourth layers of the experimental dish.

Figure 15. Top View of the First Layer (Mirrored) of the 12-Petal Experimental Design

Figure 16. Designs of the Supporting Layers of the Experimental Dish

The components of the dish were fabricated from the sheet steel using a water jet cutting. Photographs of representative parts are shown in Figure 17. Note that the fourth layer of all the petals is made as a single piece. Figure 18 shows the assembled experimental dish, sometimes referred to as the flower assembly.
Figure 17. Four Layers of One of the 12 Experimental Petals

(a) Front Side (Mirrored)

(b) Back Side

Figure 18. The Unfolded 12-Petal Dish Assembly

The experimental system for testing the dish mirrors is shown in Figure 19. In the system a nearly point light source is placed at the focal point of a precision parabolic dish. This produces an approximately collimated light source required to test solar mirrors. A translucent screen is placed at the focal plane of the mirror to enable the size of the collected energy to be measured. In the results presented, a laser was used as the light
source. Its beam was set parallel to the optical axis of the mirror and was translated along a finite set of points on the long axis of the petals. A camera measured the location reflected spot at the mirrors focal plane at these points.

Figure 19. Experimental Setup for Laboratory Mirror Testing
The results of the laboratory tests validated the effectiveness of the design concept. Typical results of these tests are shown in Figure 20. For these results, the laser is moved to 40 points evenly distributed on the long axis of a petal, covering one-half of the mirror’s aperture. It is shown for the non-optimized single layer mirror that only 16 of the 40 image points fell with the assume receiver diameter, an effective energy efficiency of 40%. For the 4-layer petal, 37 points of the 40 reflected laser beam fell within the desired focal area, an effective energy efficiency of 92.5%. These results are for the axes of the petal. Due to the small number of petals, the off-axis results for both cases would obviously be lower.
6. Summary and Conclusions
Here, a new concept for designing and fabricating large parabolic dish mirrors is presented. The dish mirror is formed from optimal-shaped flat metal petals with highly reflective surfaces. Attached to the backsides of the petals are thin metal layers that are optimized to have the reflective petals form into a parabola when their ends are pulled toward each other by cables or rods. The validity of the concept is demonstrated using Finite Element Analysis and laboratory experiments. The concept would permit flat mirror elements to be easily fabricated and efficiently packaged for shipping to field sites where they can be assembled into the parabolic dish concentrators. The concept has potential to provide precision solar parabolic solar collectors at a substantial lower cost than conventional methods.

In the design of all dish collectors such disturbances as gravity and wind loading need to be considered. This is true for this concept as well. A discussion of active compensation using servo controlled of the tensioning cable or wires and overall stiffness selection to mitigate the effects of such disturbances is the subject of our current studies and beyond the scope of this paper.

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References


